Group 15

Maldacena 3

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Maldacena
1.3.1

$$S = \int d^4x \sqrt{-g}\, -\frac{1}{2} g^{\mu\nu} (\nabla_\mu \varphi)(\nabla_\nu \varphi) \quad \Rightarrow \quad \nabla_\mu \nabla^\mu \varphi = 0$$

$$\nabla_\mu \left(-\frac{1}{2} g^{\mu\nu} \varphi \nabla_\nu \varphi \right) = -\frac{1}{2} (\nabla_\mu \varphi)(\nabla^\mu \varphi) - \frac{1}{2} \varphi \cancelto{0}{\nabla_\mu \nabla^\mu \varphi}$$

$$\Rightarrow \quad S = \int d^4x \sqrt{-g}\, \nabla_\mu \left(-\frac{1}{2} g^{\mu\nu} \varphi \nabla_\nu \varphi \right)$$

Since $\sqrt{-g}\, \nabla_\mu V^\mu = \partial_\mu (\sqrt{-g}\, V^\mu)$,

$$S = \int d^4x\, \partial_\mu \left(\sqrt{-g}\, -\frac{1}{2} g^{\mu\nu} \varphi \nabla_\nu \varphi \right)$$

$$\boxed{S = \int d^4x\, \partial_\mu \left(\sqrt{-g}\, -\frac{1}{2} \varphi\, g^{\mu\nu} \partial_\nu \varphi \right)}$$

$$S = \int d^4x\, \partial_0 (\text{ " }) + \int d^4x\, \partial_i (\text{ " })$$

$$= \int dt\, \partial_0 \left[\int d^3x \sqrt{-g}\, -\frac{1}{2} \varphi\, g^{0\nu} \partial_\nu \varphi \right]$$

$$+ \int d^3x\, \partial_i \left[\int dt \sqrt{-g}\, -\frac{1}{2} \varphi\, g^{i\nu} \partial_\nu \varphi \right]$$

$$g_{\mu\nu} = a^2(\tau)\, \eta_{\mu\nu}, \quad g^{\mu\nu} = a^{-2} \eta^{\mu\nu}, \quad \sqrt{-g} = a^4$$

$$S = \int d^4x\, \partial_\mu \left(a^2 - \frac{1}{2} \varphi\, \eta^{\mu\nu} \partial_\nu \varphi \right)$$

$$= \frac{1}{2} \int d^4x\, \partial_0 \left(a^2 \varphi\, \partial_0 \varphi \right) - \frac{1}{2} \int d^4x\, \partial_i \left(a^2 \varphi\, \partial_i \varphi \right)$$

$$= \frac{1}{2} \int dt\, \partial_0 \left(a^2 \int d^3x\, \varphi\, \partial_0 \varphi \right) - \frac{1}{2} \int dt\, a^2 \int d^3x\, \partial_i (\varphi\, \partial_i \varphi)$$

$$\varphi = \int \frac{d^3k}{(2\pi)^3}\, \varphi(k,t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$\nabla_\mu \nabla^\mu \varphi = 0 \quad \Rightarrow \quad \partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu \varphi\right) = 0$$

$$\Rightarrow \quad \partial_0\left(a^2 \dot{\varphi}\right) = a^2 \partial_i^{\,2} \varphi$$

$$2aa'\dot{\varphi} + a^2\ddot{\varphi} = a^2\partial_i^{\,2}\varphi$$

$$\ddot{\varphi} + 2\frac{\dot{a}}{a}\dot{\varphi} = \partial_i^{\,2}\varphi$$

$\Longrightarrow$ (from 1.1 a))

$$\varphi_k = A_k\,(1+i|k|t)\,e^{-i|k|t} + B_k\,(1-i|k|t)\,e^{i|k|t}$$

$$t \longrightarrow -\infty, \quad \varphi_k \sim e^{-i|k|t} \quad \Rightarrow \quad B_k = 0$$

$$\Rightarrow \quad \boxed{\varphi_k = A_k\,(1+i|k|t)\,e^{-i|k|t}}$$

$$\varphi_k(\eta_c) = \varphi_b(k) = A_k\,(1+i|k|\eta_c)\,e^{-i|k|\eta_c}$$

$$\Rightarrow \quad A_k = \frac{\varphi_b(k)}{1+i|k|\eta_c}\, e^{i|k|\eta_c}$$

$$\Rightarrow \quad \boxed{\varphi_k(\eta) = \varphi_b(k)\,\frac{1+i|k|\eta}{1+i|k|\eta_c}\, e^{-i|k|(\eta-\eta_c)}}$$

$$\boxed{\varphi(x,\eta) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, e^{-i|k|(\eta-\eta_c)}\,\frac{1+i|k|\eta}{1+i|k|\eta_c}\,\varphi_b(k)}$$

$$\partial_i \Phi = \int \frac{d^3k}{(2\pi)^3} \, i k_i \, \Phi(k,t) \, e^{ik\cdot x}$$

$$\Phi^* \partial_i \Phi = \int \frac{d^3p \, d^3k}{(2\pi)^6} \, \Phi^*(p,t) \, i k_i \, \Phi(k,t) \, e^{i(\vec{k}-\vec{p})\cdot\vec{x}}$$

$$\partial_i (\Phi^* \partial_i \Phi) = \int \frac{d^3p \, d^3k}{(2\pi)^6} \, \Phi^*(p,t) \, \Phi(k,t) \, e^{i(k-p)\cdot x} \, \vec{k} \cdot (\vec{k}-\vec{p})$$

$$\int d^3x \, \partial_i (\Phi^* \partial_i \Phi) = \int \frac{d^3p \, d^3k}{(2\pi)^3} \, \underbrace{\cancel{\vec{k}\cdot(\vec{k}-\vec{p})}}_{\to 0} \, \Phi^*(p,t) \, \Phi(k,t) \underbrace{\int \frac{d^3x}{(2\pi)^3} e^{i(k-p)\cdot x}}_{\delta^3(\vec{k}-\vec{p})}$$

$$= 0$$

$$\Rightarrow \quad \boxed{S = \frac{1}{2} \int dt \, \partial_0 \left(a^2 \int d^3x \, \Phi \, \partial_0 \Phi \right)}$$

$$S = \frac{1}{2} a^2(\eta_c) \int d^3x \, \Phi(\eta_c) \dot{\Phi}(\eta_c) - \frac{1}{2} a^2(\eta) \int d^3x \, \Phi(\eta) \dot{\Phi}(\eta) \Big|_{\eta \to -\infty}$$

$$f_k(\eta) = e^{-i|k|\eta}(1 + i|k|\eta) = e^{-i|k|\eta} + i|k|\eta \, e^{-i|k|\eta}$$

$$\dot{f}_k(\eta) = -i|k| e^{-i|k|\eta} + i|k| e^{-i|k|\eta} + k^2 \eta \, e^{-i|k|\eta}$$

$$\Phi(\eta) = \int \frac{d^3k}{(2\pi)^3} \, e^{i\vec{k}\cdot\vec{x}} \, e^{i|k|\eta_c} \, e^{-i|k|\eta} \, \frac{1 + i|k|\eta}{1 + i|k|\eta_c} \, \Phi_b(k)$$

$$\Phi(\eta) = \int \frac{d^3k}{(2\pi)^3} \, \Phi_b(k) \, e^{i\vec{k}\cdot\vec{x}} \, \frac{e^{i|k|\eta_c}}{1 + i|k|\eta_c} \, e^{-i|k|\eta} (1 + i|k|\eta)$$

$$\dot{\Phi}(\eta) = \int \frac{d^3k}{(2\pi)^3} \, \Phi_b(k) \, \frac{e^{i\vec{k}\cdot\vec{x}} e^{i|k|\eta_c}}{1 + i|k|\eta_c} \, k^2 \eta \, e^{-i|k|\eta}$$

$$S = \frac{1}{2} \int d^3x \left(a^2 \Phi \dot{\Phi} \Big|_{\eta_c} - a^2 \Phi \dot{\Phi} \Big|_{\eta \to -\infty} \right)$$

$$a^2 \Phi^* \dot{\Phi} = \frac{1}{\eta^2} \Phi^* \Phi'$$

$$= \frac{1}{\eta^2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \; \Phi_b(k) \frac{e^{i\vec{k}\cdot\vec{x}} e^{i|k|\eta_c}}{1+i|k|\eta_c} k^2 \eta \, e^{-i|k|\eta}$$

$$\Phi_b^*(p) \frac{e^{-i\vec{p}\cdot\vec{x}} e^{-i|p|\eta_c}}{1-i|p|\eta_c} e^{i|p|\eta} \, (1 - i|p|\eta)$$

$$= \frac{1}{\eta^2} \int \frac{d^3k\, d^3p}{(2\pi)^6} \frac{\Phi_b^*(p)\,\Phi_b(k)\, e^{i(|k|-|p|)\eta_c}}{(1+i|k|\eta_c)(1-i|p|\eta_c)} e^{-i(|k|-|p|)\eta} \; k^2 \eta \; (1-i|p|\eta) \; e^{i(\vec{k}-\vec{p})\cdot\vec{x}}$$

$$\int d^3x \; a^2 \Phi^* \dot{\Phi} = \frac{1}{(2\pi)^3}$$

$$\underbrace{\int \frac{d^3x}{(2\pi)^3} e^{i\vec{x}\cdot(\vec{k}-\vec{p})}}_{\delta(\vec{k}-\vec{p})}$$

$$= \frac{1}{\eta^2} \int \frac{d^3k}{(2\pi)^3} \frac{\Phi_b^*(k)\,\Phi_b(k)}{(1+i|k|\eta_c)(1-i|k|\eta_c)} k^2 \eta \; (1 - i|k|\eta)$$

$$\int d^3x \; a^2 \Phi^* \dot{\Phi} = \int \frac{d^3k}{(2\pi)^3} \frac{\Phi_b^*(k)\,\Phi_b(k)}{(1+i|k|\eta_c)(1-i|k|\eta_c)} \; k^2 \left(\frac{1}{\eta} - i|k|\right)$$

$$S = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{\Phi_b^*(k)\,\Phi_b(k)}{(1+i|k|\eta_c)(1-i|k|\eta_c)} \; k^2 \left(\frac{1}{\eta_c} \cancel{- i|k|} - \cancel{\frac{1}{\eta}}^{\,0} \cancel{+ i|k|}\right)\Bigg|_{\eta\to-\infty}$$

$$= \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{\Phi_b^*(k)\,\Phi_b(k) \; k^2}{(1+i|k|\eta_c)(1-i|k|\eta_c)} \frac{1}{\eta_c}$$

$$S = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\eta_c} \frac{\Phi_b^*(k)\,\Phi_b(k)}{1+k^2\eta_c^2} k^2$$

$d^3k = 4\pi k^2 dk$

If $\Phi_b(\vec{k}) = \Phi_b(|k|)$,

$$\frac{d^3k}{2(2\pi)^3} = \frac{k^2 4\pi\, dk}{16\pi^3} = \frac{1}{4\pi^2} k^2 dk \implies \boxed{S = \frac{1}{4\pi^2 \eta_c} \int_0^\infty dk \, k^4 \frac{\Phi_b^*(k)\Phi_b(k)}{1+k^2\eta_c^2}}$$

$\eta \to iz,$

$$S = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,\frac{1}{iz_c}\,\frac{\Phi_b^*(k)\,\Phi_b(k)\,k^2}{1-k^2 z_c^2}$$

$$iS = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,\frac{1}{z_c}\,\frac{k^2}{1-k^2 z_c^2}\,\Phi_b^*(k)\Phi_b(k) \equiv S_E$$

$$\boxed{e^{iS} = e^{-S_E}}$$

$$\frac{\delta}{\delta\Phi_b(\vec{k})}\frac{\delta}{\delta\Phi_b(\vec{k})}\Psi\Big|_{\Phi_b=0} = \frac{\delta}{\delta\Phi_b}\left(\frac{\Phi_b(k)\,k^2}{z_c(1-k^2z_c^2)}\Psi\right)\Big|_{\Phi_b=0}$$

$$= \frac{k^2}{z_c(1-k^2z_c^2)}\Psi\Big|_{\Phi_b=0} + \cancel{\Phi_b\cdots\Psi} \to 0$$

$$= \frac{k^2}{z_c(1-k^2z_c^2)}$$

As $k \to 0$, $\to 0$, so no divergence

$$\langle 0|\,T\,\Phi(\eta)\Phi(\eta')\,|BD\rangle = f^*(\eta)f(\eta') - f(\eta)f^*(\eta')$$

corresponds to the choice of contour integral

Creminelli 4

$$S_{EM} = -\tfrac{1}{4} \int d^4x \sqrt{-g}\, F_{\mu\nu} F^{\mu\nu} \quad , \quad g \equiv \det g_{\mu\nu}$$

$$F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

Since $\nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma_{\mu\nu}^{\lambda} A_\lambda$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

Write $g_{\mu\nu} = (-g)^{1/4} \tilde{g}_{\mu\nu}$, where $\det \tilde{g}_{\mu\nu} = -1$. The metric $\tilde{g}_{\mu\nu}$ is the conformally invariant part of the metric. Since the inverse metrics obey $\tilde{g}^{\mu\nu} = (-g)^{1/4} g^{\mu\nu}$,

$$S_{EM} = -\tfrac{1}{4} \int d^4x \cancel{\sqrt{-g}}\, \cancel{(-g)^{-1/4}}\, \cancel{(-g)^{-1/4}}\, F_{\mu\nu} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} F_{\alpha\beta}$$

$$= -\tfrac{1}{4} \int d^4x\, F_{\mu\nu} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} F_{\alpha\beta}$$

The action of E&M is manifestly conformally invariant.

In terms of confarmal time τ (related to proper time t by $dt = a\,d\tau$) the FRW metric can be written as

$$g_{\mu\nu} = a^2(\tau)\, \tilde{g}_{\mu\nu} ,$$

where $\tilde{g}_{\mu\nu}$ is the static metric

$$\tilde{g}_{00} = -1, \quad \tilde{g}_{0i} = \tilde{g}_{i0} = 0$$

$$\tilde{g}_{ij} = \delta_{ij} + \frac{K x^i x^j}{1 - K|\vec{x}|^2} \qquad K = +1, -1, 0$$

The field A_μ feels a static metric $\tilde{g}_{\mu\nu}$, and is thus insensitive to cosmological evolution. (Besides, I don't think photons can exist before Electroweak symmetry breaking!)

Susskind

3.1/03

3 sphere

$$R^2 = X_0^2 + X_1^2 + X_2^2 + X_3^2$$
$$= X_0^2 + \vec{x}^2$$

Ambient metric

$$ds^2 = dX_0^2 + dx_1^2 + dx_2^2 + dx_3^2$$
$$= dX_0^2 + d\vec{x}^2$$

Take $X_0 > 0$ to obtain hemisphere

$$X_0 = \sqrt{R^2 - \vec{x}^2}$$

$$dX_0 = \frac{1}{2} \frac{1}{\sqrt{R^2 - \vec{x}^2}} - 2\vec{x} \cdot d\vec{x}$$

$$= - \frac{\vec{x} \cdot d\vec{x}}{\sqrt{R^2 - \vec{x}^2}}$$

$$\Rightarrow \boxed{ds^2 = d\vec{x}^2 + \frac{(\vec{x} \cdot d\vec{x})^2}{R^2 - \vec{x}^2}}$$

Angular variables

$$\left.\begin{aligned} X_3 &= \rho \cos\theta \\ X_2 &= \rho \sin\varphi \sin\theta \\ X_1 &= \rho \cos\varphi \sin\theta \end{aligned}\right\} \quad \vec{x}^2 = \rho^2$$

$$dx_1 = \cos\varphi \sin\theta \, d\rho - \rho \sin\varphi \sin\theta \, d\varphi + \rho \cos\varphi \cos\theta \, d\theta$$

$$dx_2 = \sin\varphi \sin\theta \, d\rho + \rho \cos\varphi \sin\theta \, d\varphi + \rho \sin\varphi \cos\theta \, d\theta$$

$$dx_3 = \cos\theta \, d\rho - \rho \sin\theta \, d\theta$$

$$dx_1^2 = \cos^2\varphi \sin^2\theta \, d\rho^2 + \rho^2 \sin^2\varphi \sin^2\theta \, d\varphi^2 + \rho^2 \cos^2\varphi \cos^2\theta \, d\theta^2$$
$$- 2\rho \cos\varphi \sin\varphi \sin^2\theta \, d\rho \, d\varphi + 2\rho \cos^2\varphi \sin\theta \cos\theta \, d\rho \, d\theta$$
$$- 2\rho^2 \sin\varphi \cos\varphi \sin\theta \cos\theta \, d\theta \, d\varphi$$

$$dx_2^2 = \sin^2\varphi \sin^2\theta \, d\rho^2 + \rho^2 \cos^2\varphi \sin^2\theta \, d\varphi^2 + \rho^2 \sin^2\varphi \cos^2\theta \, d\theta^2$$
$$+ 2\rho \cos\varphi \sin\varphi \sin^2\theta \, d\rho \, d\varphi + 2\rho \sin^2\varphi \sin\theta \cos\theta \, d\rho \, d\theta$$
$$+ 2\rho^2 \sin\varphi \cos\varphi \sin\theta \cos\theta \, d\theta \, d\varphi$$

$$dx_1^2 + dx_2^2 = \sin^2\theta \, d\rho^2 + \rho^2 \sin^2\theta \, d\varphi^2 + \rho^2 \cos^2\theta \, d\theta^2$$
$$+ 2\rho \sin\theta \cos\theta \, d\rho \, d\theta$$

$$dx_3^2 = \cos^2\theta \, d\rho^2 + \rho^2 \sin^2\theta \, d\theta - 2\rho \cos\theta \sin\theta \, d\rho \, d\theta$$

$$\boxed{d\vec{x}^2 = d\rho^2 + \rho^2 \sin^2\theta \, d\varphi^2 + \rho^2 \, d\theta^2}$$

$$x_1 dx_1 = \rho \cos^2\varphi \sin^2\theta \, d\rho - \rho^2 \cos\varphi \sin\varphi \sin^2\theta \, d\varphi + \rho^2 \cos^2\varphi \cos\theta \sin\theta \, d\theta$$
$$x_2 dx_2 = \rho \sin^2\varphi \sin^2\theta \, d\rho + \rho^2 \cos\varphi \sin\varphi \sin^2\theta \, d\varphi + \rho^2 \sin^2\varphi \sin\theta \cos\theta \, d\theta$$

$$x_1 dx_1 + x_2 dx_2 = \rho \sin^2\theta \, d\rho + \rho^2 \sin\theta \cos\theta \, d\theta$$
$$x_3 dx_3 = \rho \cos^2\theta \, d\rho - \rho^2 \sin\theta \cos\theta \, d\theta$$

$$(\vec{x} \cdot d\vec{x}) = (\rho \, d\rho), \quad \boxed{(\vec{x} \cdot d\vec{x})^2 = \rho^2 d\rho^2}$$

$$ds^2 = d\rho^2 + \rho^2 \sin^2\theta \, d\varphi^2 + \rho^2 d\theta^2 + \frac{\rho^2 d\rho^2}{R^2 - \rho^2}$$
$$= \underbrace{\left(1 + \frac{\rho^2}{R^2 - \rho^2}\right)}_{R^2 / R^2 - \rho^2} d\rho^2 + \rho^2 \underbrace{\left(\sin^2\theta \, d\varphi^2 + d\theta^2\right)}_{d\Omega_2^2}$$

$$ds^2 = R^2 \left(\frac{d\rho^2}{R^2 - \rho^2} + \frac{\rho^2}{R^2} d\Omega_2^2 \right)$$

$$r \equiv \rho/R$$

$$\Rightarrow \boxed{ds^2 = R^2\left(\frac{dr^2}{1-r^2} + r^2 d\Omega_2^2\right)}$$

$$S_D = \int d^D x \sqrt{-g_0}\, R_D \cdot e^{-2\phi} \qquad g_s \sim e^{\phi}$$

$$S_4 = \int d^4 x \sqrt{-g}\, R\, V_x \qquad V_x \equiv \frac{Vol(\gamma)}{g_s^2 \alpha'^4}$$

Conformal transformation

$$\tilde{g}_{\mu\nu} \equiv \Omega g_{\mu\nu}$$

$$\sqrt{-g} = \Omega^{-d/2} \sqrt{-\tilde{g}} \qquad d=4$$

$$R = \Omega \tilde{R} - \Omega \frac{(d-1)(d-2)}{4} (\tilde{\nabla} \log \Omega)^2 + \Omega (d-1) \tilde{\Box} \log \Omega$$

$$\sqrt{-g}\, V_x R = V_x\, \Omega^{1-d/2} \sqrt{-\tilde{g}} \Big(\tilde{R} - \frac{(d-1)(d-2)}{4} (\tilde{\nabla} \log \Omega)^2 + (d-1) \tilde{\Box} \log \Omega \Big)$$

Choose $\Omega = V_x^{\frac{2}{d-2}}$

$$\sqrt{-g}\, V_x R = \sqrt{-\tilde{g}} \Big(\tilde{R} - \frac{(d-1)}{(d-2)} (\tilde{\nabla} \log V_x)^2 + 2 \frac{d-1}{d-2} \tilde{\Box} \log V_x \Big)$$

$$\frac{1}{2} \int d^4 x \sqrt{-g}\, V_x R = \frac{1}{2} \int d^4 x \sqrt{-\tilde{g}} \Big(\tilde{R} - \frac{(d-1)}{(d-2)} (\tilde{\nabla} \log V_x)^2 \Big)$$

drop bndry term

$$V_x = e^{c_x \sigma_x}$$

$$\log V_x = c_x \sigma_x$$

$$=1 \implies \boxed{c_x = \sqrt{\frac{d-1}{d-2}} = \sqrt{3/2}}$$

$$\frac{1}{2} \int d^4 x \sqrt{-g}\, V_x R = \frac{1}{2} \int d^4 x \sqrt{-\tilde{g}} \Big(\tilde{R} - \underbrace{c_x^2 \frac{(d-1)}{(d-2)}}_{=1} (\tilde{\nabla} \sigma_x)^2 \Big)$$

Silverstein (4) . 2 c.

$b = \int_\Sigma B$, gauge freedoms in $|dC_p + B \wedge dC_{p-2}|^2$, $|dB|^2$

eg. $B \to B + d\Lambda_1$, because $dd\Lambda_1 = 0$ (by definition).

looking for gauge definition for $|dC_p + B\wedge dC_{p-2}|^2$

$C_p \to C_p + \alpha_p$ (p-forms of some sort).

then $d(C_p + \alpha_p) + (B + d\Lambda_1) \wedge d(C_{p-2} + \alpha_{p-2})$

$= \underbrace{dC_p + B \wedge dC_{p-2}}_{\text{:)}} + \underbrace{(d\alpha_p + B\wedge d\alpha_{p-2} + d\Lambda_1 \wedge dC_{p-2} + d\Lambda_1 \wedge d\alpha_{p-2})}_{\to 0 \text{ (hopefully)}}$

Two possible solutions:

(I) $d\alpha_{p-2} = -dC_{p-2}$, $d\alpha_p = B\wedge dC_{p-2}$

Not exactly gauge "freedom"; & also α_p is a function of B!

(II) Recurrence relation (more generally)

$d\alpha_p = -B \wedge d\alpha_{p-2} - d\Lambda_1 \wedge d\alpha_{p-2} - d\Lambda_1 \wedge dC_{p-2}$

Given a B, a Λ_1 & C_{p-2} can build up a ladder.

$\alpha_0 = 0$, α_1 is arbitrary 1-form to initialise (??).

(I is a subset of II).

Zaldarriaga

3.1

Random Walk in 1D

$$\langle x^2 \rangle \sim t$$

$$\lambda_{\text{mean free path}} = \frac{1}{n_e \sigma_T}$$

number density of electrons

thomson scattering cross section

$$t_{mfp} = \frac{\lambda_{mfp}}{c}$$

$$N_{coll} = \frac{t_{LSS}}{t_{mfp}} = t_{LSS} \frac{c}{\lambda}$$

$$\langle x^2 \rangle = N \lambda^2 = \lambda^2 \, t_{LSS} \frac{c}{\lambda}$$

$$= \lambda_{MFP} \, c t_{LSS}$$

$$\sqrt{\langle x^2 \rangle} = \sqrt{\frac{c t_{LSS}}{n_e \sigma_T}}$$

3D $\quad \langle x^2 + y^2 + z^2 \rangle \sim \lambda_{MFP} \, c t_{LSS}$

$$\boxed{\sqrt{\langle \vec{x}^2 \rangle} = \sqrt{\frac{c t_{LSS}}{n_e \sigma_T}}}$$

3.2

$$\sigma_T = \frac{8\pi}{3}\left(\frac{q^2}{4\pi\varepsilon_0 m_e c^2}\right)^2$$

$$= 6.65 \times 10^{-25}\ \text{cm}^2$$

$$t_{LSS} \sim 400{,}000 \text{ years}$$

$\cancel{N_e \sim 10^{78}}$ $\cancel{n_\gamma = 411 \frac{1}{cm^3}}$

$\cancel{n_e \sim 10^{78}}$ ~~γ to b ratio~~

$$n_\gamma = \frac{N_\gamma}{V_0} = 411/\text{cm}^3$$

$$\frac{4110}{2} = 5 + 50 + 2000 = 2055 = 2\times10^3$$

$$n_b = 5\cdot 10^{-10}\, n_\gamma = 2\times 10^{-7}/\text{cm}^3$$

Assume $n_e = n_b$

$$n_e = 2\times10^{-7}/\text{cm}^3$$

$$n_e\big|_{z\sim1100} = 1100^3\, n_e\big|_0$$

$$= 270\ /\text{cm}^3$$

$$\sim 3\times10^2\ \text{cm}^{-3}$$

$$n_e\sigma_T = 1.8\times10^{-22}\ \text{cm}^{-1}$$

$$\boxed{\lambda = \frac{1}{n_e\sigma_T} \sim 6\times10^{21}\ \text{cm}}$$

$$c\,t_{LSS} \sim 4\times10^{21}\ \text{m}$$

$$\Rightarrow \boxed{\sqrt{\langle \vec{x}^2\rangle} \sim 5\times10^4 \text{ light years}}$$