(1.3) MALDACENA Q.3 GROUP 20 (1)

Suppose i want to fix boundary conditions.

$$\phi_k = \phi_k^0 \frac{(1+ik\eta)\, e^{-ik\eta}}{(1+ik\eta_c)\, e^{-ik\eta_c}}$$

$$S = \int d^4x \sqrt{-g}\, -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = \int d^4x \frac{1}{2\eta^2}(\dot\phi^2 - \nabla\phi^2) =$$

$$\cancel{= \int dt \frac{d^3k}{(2\pi)^3} \frac{1}{2\eta^2} (\dot\phi_k \dot\phi}$$

$$\int dt\, d^3x \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3} \frac{1}{2\eta^2}(\dot\phi_k \dot\phi_{k'} + k\cdot k' \phi_k \phi_{k'})\, e^{i(k+k')x} =$$

$$= \int dt \frac{d^3k}{(2\pi)^3} \frac{1}{2\eta^2}(\dot\phi_k \dot\phi_{-k} - k^2 \phi_k \phi_{-k}) =$$

$$\text{EOM}: \quad 4\left(-\frac{d}{dt} \frac{1}{\eta^2} \dot\phi_{-k} - k^2 \phi_{-k}\right) = 0.$$

$$= \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{0} dt \frac{1}{2}\frac{d}{dt} \frac{1}{\eta^2} \dot\phi_{+k}\, \phi_{-k} + \text{EOM} =$$

$$= \int \frac{d^3k}{(2\pi)^3} \cancel{\phi_{-k}\left(\frac{1}{2\eta^2}\phi_k\right)}\ \phi_{-k} \frac{1}{2\eta^2} \dot\phi_k \Big|_{-\infty}^{0 \to \eta_c} =$$

$$\cancel{= \int \frac{d^3k}{(2\pi)^3} \phi^0_{-k} \frac{1}{2\varepsilon^2} \frac{i k^2 \varepsilon}{1+ik\eta_c} \frac{\phi_k}{e^{-ik\eta_c}}}$$

$$= \int \frac{d^3k}{(2\pi)^3} \phi^0_{-k} \frac{1}{2\eta_c} \frac{k^2}{1-ik\eta_c} \phi^0_k . \quad \underline{\text{ok}}$$

Euclidean AdS $\qquad ds^2 = \frac{d\eta^2 + dx^2}{\eta^2}$

$$S_E = \int d^4x \sqrt{-g}\ \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$$\partial_\mu \sqrt{g}\, g^{\mu\nu}\partial_\nu\phi = 0$$

$$\partial_0 \frac{1}{\eta^2}\partial_0\phi + \frac{1}{\eta^2}(\nabla\phi)^2 = \frac{\ddot\phi}{\eta^2} - \frac{2}{\eta^3}\dot\phi - \frac{k^2\phi}{\eta^2} = 0.$$

$$\ddot\phi - \frac{2}{\eta}\dot\phi - k^2\phi = 0$$

$$\phi = (1\pm k\eta)\, e^{\mp k\eta}$$

$$\dot\phi = \pm k e^{\mp k\eta} \mp k(1\pm k\eta)e^{\mp k\eta} = -k^2\eta\, e^{\mp k\eta}$$

$$\ddot\phi = -k^2 e^{\mp k\eta} \pm k^3\eta\, e^{\mp k\eta}$$

$$-k^2 \pm k^2\eta - \frac{2}{\eta}(-k^2\eta) - k^2(1\pm k\eta) =$$

$$= -k^2 \pm k^3\eta + 2k^2 - k^2 \mp k^3\eta = 0.$$

I consider the solution which decays at $\eta = \mp\infty$

$$\phi_k^E = \phi_k^{0E}\,\frac{(1\pm k\eta)}{(1\pm k\eta_c)}\,\frac{e^{\mp k\eta}}{e^{\mp k\eta_c}} \qquad \dot\phi_k^E = -\phi_k^{0E}\,\frac{k^2\eta}{1+k\eta_c}\,\frac{e^{-k\eta}}{e^{-k\eta_c}}$$

$$S_E = \int\frac{d^3k}{(2\pi)^3}\,\phi^E_{-k}\,\frac{1}{2\eta^2}\,\dot\phi_k\Big|_{\eta_c}^{\infty} = +\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{2\eta_c}\,\frac{k^2}{(1+k\eta_c)}\,\phi^{0E}_{-k}\phi^{0E}_k$$

$$\eta \to -i\eta$$

$$S_E \to -\int\frac{d^3k}{(2\pi)^3}\,\frac{i}{2\eta_c}\,\frac{k^2}{1-ik\eta_c}\,\phi^0_{-k}\phi^0_k = -i\,S_{dS}$$

$$\Rightarrow -S_{EAdS} \to i\,S_{dS}$$

$$ds^2 \to \frac{-d\eta^2 + dx^2}{-\eta^2} \Rightarrow R \to iR \qquad \text{if}\ \left[\begin{array}{l}\eta_{dS} \to i\,\eta_{EAdS}\\ R_{dS} \to i\,R_{EAdS}\end{array}\right]$$

Consider dS_4 $\qquad \eta_c \sim 0$ $\qquad$ 3

$$S = \int \frac{d^3k}{(2\pi)^3} \phi^0_{-k}\phi^0_k \frac{k^2}{2\eta_c(1-ik\eta_c)} = \int \frac{d^3k\, d^3k'}{(2\pi)^{3\cdot 2}} \delta^3(k+k') \frac{k^2}{2\eta_c(1-ik\eta_c)} \phi_k \phi_{k'}$$

$$\frac{\delta}{\delta\phi(k)_1} \frac{\delta}{\delta\phi(k)_2}{}_{\text{boundary}}\, e^{iS} = (2\pi)^3 \delta(k+k') \frac{ik^2}{\eta_c(1-ik\eta_c)}$$

$$\frac{ik^2}{\eta_c(1-ik\eta_c)} \simeq \underbrace{\frac{ik^2}{\eta_c}}_{\text{local}} + k^3 + \dots$$

$k^3 \to$ non local

$\frac{ik^2}{\eta_c}$ $\leftarrow$ regularize.

$$\langle BD|\phi(x)\phi(y)|BD\rangle = \langle BD| \int \frac{d^3k\, d^3k'}{(2\pi)^{3\cdot 2}} f^+_k(x) e^{-ikx} f_{k'}(y) e^{ik'y} \underbrace{a_k a^+_{k'}}_{(2\pi)^3\delta(k-k')} |BD\rangle$$ 4

Assuming also different conformal times

$$\int \frac{d^3k}{(2\pi)^3} \frac{1-ik\eta}{\sqrt{2}k^{3/2}} e^{ik\eta} \frac{1+ik\eta'}{\sqrt{2}k^{3/2}} e^{-ik\eta'} e^{-i\vec{k}\cdot\vec{x}} e^{+i\vec{k}\cdot\vec{y}} =$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{(1-ik\eta)(1+ik\eta')}{2k^3} e^{ik(\eta-\eta')} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}.$$

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = R^2.$$

$$X_0 = \operatorname{sh} t + \frac{1}{2}\vec{x}^2 e^t = -\frac{1/\eta + \eta}{2} - \frac{1}{2}\vec{x}^2\frac{1}{\eta} = -\frac{1}{2\eta}(1+\eta^2+\vec{x}^2)$$

$$X_1 = \operatorname{ch} t - \frac{1}{2}\vec{x}^2 e^t = -\frac{1/\eta - \eta}{2} + \frac{1}{2}\vec{x}^2\frac{1}{\eta} = -\frac{1}{2\eta}(1+\eta^2-\vec{x}^2)$$

In flat slicing the invariant ~~mean~~ interval is

$$\cosh(t_1 - t_2) - \frac{1}{2}(\vec{x}_1 - \vec{x}_2)^2 e^{t_1 + t_2}$$ (Non sono sicuro)

conformal metric has $\eta = -e^{-t}$

$$\cosh t_1 - t_2 = \frac{e^{t_1 - t_2} + e^{-t_1 + t_2}}{2} = \frac{e^{t_1} e^{-t_2}}{2} - \frac{e^{-t_1} e^{t_2}}{2} =$$

$$= \frac{1}{2}\left(\frac{\eta_2}{\eta_1} + \frac{\eta_1}{\eta_2}\right)$$

Interval $= \cancel{\frac{1}{2}\left(\frac{\eta_2}{\eta_1} \mp \frac{\eta_1}{\eta_2}\right) - \frac{1}{2}(\vec{x}_1 - \vec{x}_2)^2 \frac{1}{\eta_1 \eta_2}}$

$$= \frac{1}{2\eta_1\eta_2}\left[(\eta_2 \mp \eta_1)^2 - (\vec{x}_1 - \vec{x}_2)^2\right]$$ OK

This is the invariant dS intervall.

The final result of the dS calculation will have to be a function of this d-intervall. The function in this form must be finite.

CREMINELLI Q. 4. GROUP 20 ①

Scale invariance

Scalar field : $\phi'(x') = \lambda^\alpha \phi(x)$ $\quad x' = \lambda x$

$$\Rightarrow \phi'(x') = \lambda^\alpha \phi(\lambda^{-1} x)$$

$$S = \int d^4x \, \frac{1}{2} \partial_\mu\phi \, \partial_\mu\phi \longrightarrow \int d^4x \, \frac{1}{2} \lambda^{2\alpha} \partial_\mu \phi(\lambda^{-1}x) \partial_\mu \phi(\lambda^{-1}x) =$$

$$\lambda^{-1} x = y \Rightarrow \quad = \int d^4y \, \frac{1}{2} \lambda^{2\alpha+4-2} \partial_\mu\phi(y)\partial_\mu\phi(y)$$

$$\Rightarrow 2\alpha + 4 - 2 = 0 \Rightarrow \boxed{\alpha = -1}$$

Metric

$$g'_{\mu\nu}(x') = g_{\alpha\beta}(x) \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} = \lambda^{-2} g_{\alpha\beta}(x)$$

$$g'_{\mu\nu}(x) = \lambda^{-2} g_{\alpha\beta}(\lambda^{-1} x).$$

Notice that since $\sqrt{g}\, g^{\mu\nu} \longrightarrow \lambda^{-4}\lambda^{2} = \lambda^{-2} \neq \lambda^0$ the action of a scalar field ~~as~~ coupled with gravity is not scale invariant.

Vector field $\quad A'_\mu(x) = \lambda^{-1} A_\mu(\lambda^{-1} x)$

$$S = \int d^4x \, \sqrt{-g}\, g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \longrightarrow$$

$$\longrightarrow \lambda^4 \lambda^{-4} \lambda^2 \lambda^2 \lambda^{-1} \lambda^{-1} \lambda^{-1} \lambda^{-1} \, S = S.$$

~~Thus~~ Scale invariance bnt conformal invariance.

Then at ~~the~~ first order in perturbation the EOM of the vector field are the same as in Minkowski, so there is no particle creation.

Group 20 - Solutions to Susskind and (part of) Silverstein

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I. SUSSKIND PROBLEM 3

The space-like part of the metric is given by:

$$\frac{R}{1-r^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

Making the substitution

$$\frac{dr}{\sqrt{1-r^2}} = d\chi \rightarrow \sin r = \chi, \tag{2}$$

we have that the space-like part becomes

$$R^2\left(d\chi^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{3}$$

$$\rightarrow R^2\left(d\chi^2 + \sin^2\chi^2 d\theta^2 + \sin^2\chi\sin^2\theta d\phi^2\right) \tag{4}$$

which is a 3-sphere. In addtion from Eq. (2) we have that $0 < r < 1$, hence $0 < \chi < \pi/2$, so we are restricted to a hemisphere.

II. SILVERSTEIN PROBLEM 2

A. Part a)

We parameterise the spherically symmetric D-4 dimensional compactification volume by its size L which varies along the the 4D space as $L(x)$. We start with the metric

$$ds^2 = \widetilde{g_{\mu\nu}(x)}dx^\mu dx^\nu + L^2(x)\widetilde{g_{ij}(y)}dx^i dx^j. \tag{5}$$

We rescale the metric such that $g^D_{AB} = L^2(x)g^D_{AB}$, hence the line element becomes

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + L^2(x)g_{ij}(y)dx^i dx^j. \tag{6}$$

From the above and Wald we then have a relation for the Einstein term as a function of the individual components:

$$\widetilde{\mathcal{R}} = L^{-2}(x)\left\{\mathcal{R} - 2(D-1)g^{\mu\nu}\nabla_\mu\nabla_\nu \ln L(x)\right\} \tag{7}$$

The action is then

$$\begin{aligned} S_E &= \int\sqrt{-g}\mathcal{R} && (8)\\ &= L^{-2}\left\{\left(\mathcal{R}^4(L^{-2}g_{\mu\nu}) + \mathcal{R}^{D-4}(g_{ij})\right)\right. && (9)\\ &\quad - 2(D-1)\left(L^2 g^{\mu\nu}\partial_\mu(L^{-2}g_{\mu\nu})\partial_\nu \ln L\right) && (10)\\ &\quad \left. - 2(D-1)\left(L^2 g^{\mu\nu}\partial_\mu \ln L\partial_\nu(L^{-2}g_{\mu\nu})\right)\right\} && (11)\end{aligned}$$

Now using

$$g^{\mu\nu}\nabla_\mu\nabla_\nu f = \frac{1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu f\right), \tag{12}$$

we can write the second last term as

$$(L^2 g^{\mu\nu}\partial_\mu(L^{-2}g_{\mu\nu})\partial_\nu \ln L) = \frac{L^4}{\sqrt{-g}}\left(L^2 g^{\mu\nu}\frac{\sqrt{-g}}{L^4}\partial_\nu \ln L\right), \tag{13}$$

and similarly for the last term.
We also note that

$$\mathcal{R}^4(L^2 g_{\mu\nu}) = L^2\left\{\mathcal{R}^4 g_{\mu\nu} + 6g^{\mu\nu}\partial_\mu\partial_\nu \ln L - 6g^{\mu\nu}\partial_\mu \ln L\partial_\nu \ln L\right\} \tag{14}$$

With further simplifications, the final action is given by:

$$S = \int d^4x \int d^{D-4}y\sqrt{-g^4}\sqrt{-g^{D-4}}L^{D-4}\left\{R^4(g_{\mu\nu}) + L^{-2}R^{D-4}(g_{ij} + (D-4)(D-5)\partial_\mu \ln L\partial_\nu \ln L)\right\} \tag{15}$$

Given that we require $L > 0$ to avoid a singularity, we can write $L = e^{\sigma_T/M_p}$. However, by considering the term L^{D-4} in the action, this results in $V \propto e^{c_X\sigma_X/M_p}$ where $c_X = D - 4$ is not small.

SILVERSTEIN ①

alternative to 2a).

GROUP 20

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + R^2(x) \left[dS_x^2 = \tilde{g}^{IJ} dX_I dX_J \right]$$

$$\Gamma^M_{NR} = \frac{1}{2} g^{MA} \left(g_{AN,R} + g_{AR,N} - g_{RN,A} \right)$$

$$\Gamma^\mu_{\nu\lambda} = \Gamma^{\mu\,(4)}_{\nu\lambda} \qquad \text{Depends only on } x^\mu$$

$$\Gamma^I_{J\mu} = \partial_\mu \log R \, \delta^I_J \qquad \text{Depens only on } x^\mu$$

$$\Gamma^I_{JK} = \Gamma^{I\,(6)}_{JK} \quad , \quad \Gamma^\mu_{IJ} = -\frac{1}{2} g^{\mu\nu} \partial_\nu R^2 \, \tilde{g}_{IJ}$$

$$R = g^{\mu\nu} R_{\mu\nu} + g^{\mu I} R_{\mu I} + g^{IJ} R_{IJ} = g^{\mu\nu} R_{\mu\nu} + g^{IJ} R_{IJ} =$$

$$= g^{\mu\nu} R_{\mu\nu} + \tilde{g}^{IJ} R_{IJ} R^{-2}$$

$$R_{MN} = R_{MAN}{}^A$$

$$R_{MNR}{}^S = \Gamma^S_{MR,N} - \Gamma^S_{NR,M} + \Gamma^A_{MR} \Gamma^S_{AN} - \Gamma^A_{NR} \Gamma^S_{AM}.$$

$$R_{\mu I\nu}{}^I = \cancel{\Gamma^I_{\mu\nu,I}} - \Gamma^I_{I\nu,\mu} + \Gamma^A_{\mu\nu} \Gamma^I_{AI} - \Gamma^A_{I\nu} \Gamma^I_{A\mu} =$$

$$= - \Gamma^I_{I\nu,\mu} + \Gamma^\alpha_{\mu\nu} \Gamma^I_{\alpha I} - \Gamma^J_{I\nu} \Gamma^I_{J\mu}. = ①$$

$$R_{I\alpha J}{}^\alpha = \Gamma^\alpha_{IJ,\alpha} - \cancel{\Gamma^\alpha_{\alpha J,I}} + \Gamma^A_{IJ} \Gamma^\alpha_{A\alpha} - \Gamma^A_{\alpha J} \Gamma^\alpha_{AI} =$$

$$= \Gamma^\alpha_{IJ,\alpha} + \Gamma^\beta_{IJ} \Gamma^\alpha_{\beta\alpha} - \Gamma^K_{\alpha J} \Gamma^\alpha_{KI} = ②$$

$$R_{\mu\alpha\nu}{}^\alpha = R^{(4)}_{\mu\nu} \quad , \quad R_{IKJ}{}^K = R^{(x)}_{IJ}$$

$$R_{\mu\nu} = R^{(4)}_{\mu\nu} - (D-4) \partial_\mu \partial_\nu \log R + \Gamma^\alpha_{\mu\nu} (D-4) \partial_\alpha \log R$$
$$- (D-4) \partial_\mu \log R \, \partial_\nu \log R.$$

$$R_{IJ} = R^{(D-4)}_{IJ} - \frac{1}{2} \tilde{g}_{IJ} \partial_\alpha (g^{\alpha\beta} \partial_\beta R^2) \mp \frac{1}{2} \tilde{g}_{IJ} \Gamma^\alpha_{\alpha\beta} g^{\beta\lambda} \partial_\lambda R^2$$
$$+ \frac{1}{2} \tilde{g}_{IJ} \partial_\alpha \log R \, g^{\alpha\lambda} \partial_\lambda R^2.$$

$$M_p^{D-2} \int \sqrt{g^4}\, A_{S^{D-4}}\, R^{D-4} \left[\left(R^{(4)}_{Ricci} + R^{-2} R^{D-4}_{Ricci} \right) + 2(D-4)(D-5) (\partial_\mu \log R)^2 \right]$$

$M_p = 1$

$R = e^{\rho/M_p}$

The lagrangian for the ρ (neglecting the dilaton is)

$$\int \sqrt{g}\, \underbrace{A_{S^{D-4}}\, e^{(D-4)\frac{\rho}{M_p}}}_{M_p^2} \left[R^{(4)}_{Ricci} + e^{-2\rho/M_p} R^{D-4}_{Ricci} + 4(D-4)(D-5) \frac{(\partial_\mu \rho)^2}{2} \right]$$

$$\rho \to \frac{\rho}{2\sqrt{(D-4)(D-5) A_{S^{D-4}}}} \quad \frac{1}{e^{\frac{D-4}{2}\rho_0}}$$

$$= \cancel{\int \sqrt{g^4} \left\{ \frac{1}{2} (\partial_\mu \rho)^2 \right\}} \; A_{S^{D-4}} \; \cancel{e^{\frac{(D-4)\rho}{2\sqrt{(D-4)(D-5)}A_{S^{D-4}}} e^{-\frac{D-4}{2}\rho_0}}}$$

$$\cancel{(R \times e^{xD4\rho})}$$

$$\cancel{\int \sqrt{g} \; M_p^2 \rho R^{(4)}_{Ricci} = \frac{1}{2} M_p^2}$$

Notice that $V_\Lambda = A_{S^{D-4}} R^{D-4} = A_{S^{D-4}}\, e^{\frac{1}{2}\sqrt{\frac{D-4}{D-5}}\frac{\rho}{M_p}}$ in terms of the canonically normalized ρ.

The whole lagrangian is multiplied by M^{D-2}

$$M_p^2 = M^{D-2} R^{D-4} A_{S^{D-4}}\, e^{(D-4)\rho_0}$$

$$* = M_p^2 \int \sqrt{g} \left[R^{(4)}_{Ricci} + e^{-2\rho_0} R^{(D-4)}_{Ricci} + 4(D-4)(D-5) \frac{(\partial_\mu \rho)^2}{2} \right] e^{(D-4)\rho}$$

$\langle \rho \rangle = 0$

$\rho \to \rho / M_p$

powers of ρ

$$M_p^2 \int \sqrt{g}\, R^{(4)}_{Ricci} + \cdots + M_p^2 \int \sqrt{g}\, e^{-2\rho_0} R^{(D-4)}_{Ricci}\, e^{(D-4)\rho/M_p} +$$

$$M_p^2 \int \sqrt{g}\, e^{(D-4)\rho} R^{(4)}_{Ricci} \qquad + 4(D-4)(D-5) \frac{\partial_\mu \rho}{2} e^{(D-4)\rho}$$

$$V = \int \sqrt{g}\, \underbrace{M_p^2 e^{-2\rho} R^{(D-4)}_{Ricci}\, e^{\frac{\sqrt{D-4}}{2(D-5)}\rho/M_p}}$$

↳ Will be 0 if the X space is flat (torus)

In no way $\frac{\sqrt{D-4}}{2(D-5)}$ is small.

silverstein (2b)

A negative mass Schwarzschild BH has metric

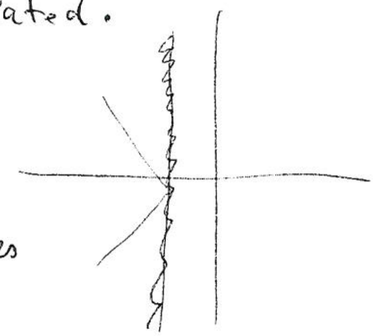
$$ds^2 = -\left(1 + \frac{2GM}{r}\right)dt^2 + \left(1 + \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2$$

there's no horizon and a naked singularity.

Since ~~$T^{00} = -m\,\delta(x)$~~ & the ~~NEC~~ NEC is violated.

the mass is negative

It will be possible to violate the second law of thermodynamics throwing negative ~~pa~~ mass particles into BH.



~~I have no idea what an orientifold is.~~ * 22/11 23.20 ; Eva explained

(2c) $|dC_p + B \wedge dC_{p-2}|^2$

$B \to B + d\Lambda_1$

$C_p \to C_p - d\Lambda_1 \wedge C_{p-2}$

$dC_p \to dC_p - d\Lambda_1 \wedge dC_{p-2}$

$B \wedge dC_{p-2} \to B \wedge dC_{p-2} + d\Lambda_1 \wedge dC_{p-2}$

they cancel to have a gauge invariance

2-d) Silverstein

I consider the metric

$$ds^2 = \frac{r^2}{R^2}\tilde{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + ds^2_X$$

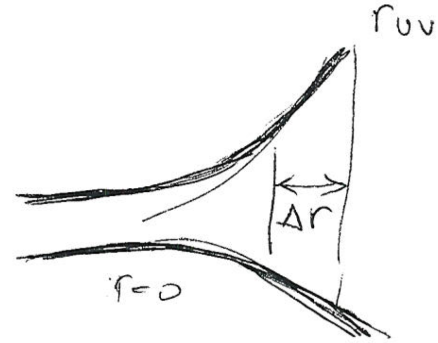
where $\dim(X) = 5$.

The 10-D action is $\quad (M^8 = 2/\alpha'^4)$

$$S_{10} = \frac{M^8}{2}\int d^{10}x\sqrt{g^{10}}\,R \quad =$$

The christoffel symbols are given by

$$\Gamma^M_{NR} = \frac{1}{2}g^{MA}(g_{AN,R} + g_{AR,N} - g_{RN,A}).$$



Notice that

$\Gamma^\mu_{\nu\rho} = \tilde{\Gamma}^\mu_{\nu\rho} + \ldots$ where $\tilde{\Gamma}$ is the symbol associated with $\tilde{g}$.

This implies that $\text{Riemann}_{\mu\nu\rho}{}^{\sigma} = \widetilde{\text{Riemann}}_{\mu\nu\sigma}{}^{\rho} + \ldots$

and $R_{\mu\nu} = \tilde{R}_{\mu\nu} + \ldots$

$$R = g^{\mu\nu}R_{\mu\nu} + \ldots = \frac{R^2}{r^2}\tilde{g}^{\mu\nu}\tilde{R}_{\mu\nu} + \ldots$$

$$= \frac{M^8}{2}\int d^4x\int_0^{r_{UV}} dr\,\sqrt{\tilde{g}^4}\,\frac{r^3}{R^3}\frac{R^2}{r^2}\tilde{R} + \ldots =$$

$$= \frac{M^8}{2}\mathrm{Vol}(X)\frac{r_{UV}^2}{2R}\int d^4x\sqrt{\tilde{g}^4}\,\tilde{R} + \ldots$$

$$M_{pl}^2 = M^8\,\mathrm{Vol}(X)\,\frac{r_{UV}^2}{2R}.$$

The Lyth bound states that the tensor-scalar ratio r is smaller than $r < \frac{4\pi\Delta\varphi^2}{M_{pl}^2}$, in particular

$$r < \frac{4\pi\Delta\varphi^2}{M_p^2} = \frac{4\pi\Delta\varphi^2 R\alpha'^4}{\mathrm{Vol}(X)\,r_{UV}^2} < \frac{4\pi\alpha'^2 R}{\mathrm{Vol}(X)} \quad \text{using } \Delta\varphi < \frac{r_{UV}}{\alpha'}.$$