④

Maldacena 1.4

$$ds^2 = d\rho^2 + \sinh^2\rho \, d\Omega_3^2$$

$$\int \sqrt{g} = 2\pi^2 \int \sinh^3\rho \, d\rho = 2\pi^2 \left(\frac{1}{3}\cosh^3\rho_c - \cosh\rho_c + 2/3 \right)$$

metric on bdy: $ds^2 = \sinh^2\rho_c \, d\Omega_3^2$

$$\partial_\rho h_{ab} = \sinh 2\rho \, d\Omega_3^2$$

$$\frac{1}{2} h^{ab} \partial_\rho h_{ab} = \frac{3}{\tanh\rho}$$

$$\Rightarrow \int_{\partial\Sigma_4} K \sqrt{-g_B} = 2\pi^2 \times \frac{3}{\tanh\rho_c} \cdot \sinh^3\rho_c = 6\pi^2 \sinh^3\rho_c \cosh\rho_c$$

$$\Rightarrow S_E = \frac{R_{AdS}^2}{16\pi G_N} \left\{ -4\pi^2 \left(\frac{1}{3}\cosh^3\rho_c - \cosh\rho_c + 2/3 \right) - 12\pi^2 \sinh^3\rho_c \cosh\rho_c \right\}$$

Keeping finite part give

$$\Psi = e^{+\pi R_{AdS}^2 / 6G_N}$$

Creminelli 3 group 4

2.3 ACTION FOR A SCALAR FIELD $\varphi(\bar{x},t)$ WITH POTENTIAL $V(\varphi)$ IN A GENERAL 4D SPACETIME:

$$S = \frac{1}{2} M_p^2 \int d^4x \sqrt{-g}\left[R - \partial_\mu\varphi\partial^\mu\varphi - 2V(\varphi)\right]$$

PURE DE SITTER AND MASSLESS:

$$\ddot{\varphi} + 3H\dot{\varphi} - \partial_i\partial^i\varphi = 0$$

FOR PERTURBATIONS $\delta\varphi(\bar{x},t)$ WHERE $\varphi(\bar{x},t) \to \varphi(t) + \delta\varphi(\bar{x},t)$:

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} - \partial_i\partial^i\delta\varphi = 0$$

IN FOURIER SPACE:

$$\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k - \frac{k^2}{a^2}\delta\varphi_k = 0$$

TO GET IN FORM OF S.H.O:

DEFINE CONFORMAL TIME: $dt = a\,d\tau \to \tau = -\frac{1}{aH}$

$$\delta\varphi_k'' + 2Ha\,\delta\varphi_k' + k^2\delta\varphi = 0 \qquad ' \equiv \frac{\partial}{\partial\tau}$$

DEFINE NEW VARIABLE: $v_k = a\,\delta\varphi_k$:

$$v_k'' + \left(k^2 - \frac{a''}{a}\right)v_k = 0$$

$$a = -\frac{1}{H\tau} \to \frac{a''}{a} = \frac{2}{\tau^2}$$

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right)v_k = 0$$

EXACT SOLUTION IS GIVEN BY:

$$V_k = \frac{A e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + \frac{B}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)$$

NEED THIS TO REDUCE TO THE ZERO POINT FLUCTUATION OF A FREE FIELD IN FLAT SPACETIME WHEN ON SCALES CORRESPONDING TO: $|k\tau| >> 1$:

$$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}$$

HENCE:

$$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

AND

$$\delta\varphi_k = \frac{1}{a}\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

TWO POINT FUNCTION:

$$\langle \delta\varphi_k, \delta\varphi_{k'} \rangle = (2\pi)^3 \delta^3(\bar{k} + \bar{k}')\,|\delta\varphi_k|^2$$

$$= (2\pi)^3 \delta^3(\bar{k} + \bar{k}')\,\frac{1}{2k}\left(1 + \frac{1}{(k\tau)^2}\right)\frac{1}{a^2}$$

$$= (2\pi)^3 \delta^3(\bar{k} + \bar{k}')\,\frac{1}{2k^3 a^2 \tau^2}\left(1 + k^2\tau^2\right)$$

$$= (2\pi)^3 \delta^3(\bar{k} + \bar{k}')\,\frac{H^2}{2k^3}\left(1 + k^2\tau^2\right)$$

SUPERHORIZON SCALES: $|k\tau| \ll 1$

$$\langle \delta\varphi_k \delta\varphi_{k'} \rangle = (2\pi)^3 \delta^3(\bar{k}+\bar{k}') \frac{H^2}{2k^3}$$

TWO POINT CORRELATION FUNCTION IS THE FOURIER TRANSFORM OF THE POWER SPECTRUM, SO:

$$\xi(x) \sim H^2 \int \frac{1}{k^3} e^{ikx} d^3k$$

WHICH DIVERGES FOR $k=0$. (IR)

IR DIVERGENCE IS BECAUSE WE ARE IN PURE DE-SITTER SPACE. ~~[illegible]~~ MORE REALISTIC CALCULATION WOULD BE QUASI-DE SITTER WHERE IR DIVERGENCE DOES NOT OCCUR.

Group 4.

Susskind #4.

$$P_a(n+1) = P_a(n) \underbrace{- \sum_b \gamma_{ba} P_a(n) + \sum_b \gamma_{ab} P_b(n)}_{\text{transitions}} \qquad (*)$$

$$= \sum_b \delta_{ab} P_b(n) + \sum_b \underbrace{\left(- \sum_c \gamma_{ca} \delta_{ab} + \gamma_{ab} \right)}_{} P_b(n)$$

$$\Gamma_{ab} = - \delta_{ab} \sum_c \gamma_{ca} + \gamma_{ab} .$$

From detailed balance,

$$\gamma_{ab} n_b = \gamma_{ba} n_a ,$$

$$\Rightarrow \frac{\gamma_{ab}}{\gamma_{ba}} = \frac{n_a}{n_b} = e^{S_a - S_b}$$

$$\Rightarrow \underset{\displaystyle M_{ab}}{\underset{|||}{\gamma_{ab} e^{S_b}}} = \underset{\displaystyle M_{ba}}{\underset{|||}{\gamma_{ba} e^{S_a}}}$$

Let $P_a = e^{S_a/2} \phi_a$, transition equation $(*)$ becomes

$$\phi_a(n+1) = \phi_a(n) - \sum_c M_{ca} e^{-S_a} \phi_a(n) + e^{-S_a/2} \sum_b M_{ab} e^{-S_b} e^{S_b/2} \phi_b(n)$$

$$= \phi_a(n) + \sum_b \underbrace{\left(- \sum_c \delta_{ab} M_{ca} e^{-S_a} + e^{-\frac{S_a}{2}} M_{ab} e^{-\frac{S_b}{2}} \right)}_{\substack{\text{transition matrix} \\ \text{manifestly symmetric}}} \phi_b(n)$$

transition matrix

manifestly symmetric.

zero mode $\phi_a \propto e^{\frac{S_a}{2}}$:

$$0 \overset{?}{=} \sum_b \left(-\delta_{ab} \sum_c M_{ac} e^{-S_a} + e^{-\frac{S_a}{2}} M_{ab} e^{-\frac{S_b}{2}} \right) e^{\frac{S_b}{2}}$$

$$= -\sum_c M_{ac} e^{-S_a} e^{\frac{S_a}{2}} + \sum_b e^{-\frac{S_a}{2}} M_{ab} = 0$$

all other eigenvalues are non-positive :

let them be λ_i, then in equation (*)

the full transition matrix for P_a is

$$\delta_{ab} + \Gamma_{ab}$$

The eigen values are $1 + \lambda_i$.

But since probability is conserved,

the full transition matrix must have

$$1 + \lambda_i \leq 1 \quad , \qquad i = 1, 2, \cdots$$

otherwise $\left(\delta_{ab} + \Gamma_{ab} \right)^n \sim (1+\lambda_i)^n$ diverges.

Therefore

$$\lambda_i \leq 0 .$$

Silverstein 1

4.1 a) $V(\varphi) = \mu^{4-p}\lambda\varphi^{p} \qquad 0 < p \leq 2$

$$N = \int_{t}^{t_E} H\,dt' = \int_{\varphi}^{\varphi_E} \frac{H}{\dot\varphi'}\,d\varphi' = \int_{\varphi}^{\varphi_E} \frac{3H^2}{3H\dot\varphi}\,d\varphi'$$

Slow Roll: $3H\dot\varphi \simeq -V_{,\varphi}$

$3H^2 m_p^2 \simeq V$

$$N \simeq -\int_{\varphi}^{\varphi_E} \frac{V}{m_p^2 V_{,\varphi'}}\,d\varphi'$$

$$V_{,\varphi} = p\mu^{4-p}\lambda\varphi^{p-1}$$

$$\frac{V}{V_{,\varphi}} = \frac{\varphi}{p}$$

$$N = \int_{\varphi_E}^{\varphi} \frac{\varphi'}{p m_p^2}\,d\varphi' = \frac{1}{2p m_p^2}\left(\varphi^2 - \varphi_{EA}^2\right)$$

$$\epsilon = \frac{1}{2} m_p^2 \left(\frac{V_{,\varphi}}{V}\right)^2 = \frac{1}{2} m_p^2 \frac{p^2}{\varphi^2}$$

$$\epsilon_E \simeq \frac{1}{2} m_p^2 \frac{p^2}{\varphi_E^2} \simeq 1 \rightarrow \varphi_E \simeq \sqrt{\frac{1}{2} m_p^2 p^2} = \frac{1}{\sqrt{2}} m_p p$$

$$N = \frac{1}{2p m_p^2}\left(\varphi^2 - \frac{1}{2} m_p^2 p^2\right) = \frac{\varphi^2}{2p m_p^2} - \frac{p}{4}$$

$$N = \frac{1}{2p m_p^2}\varphi^2 - \frac{p}{4}$$

$N \gtrsim 60$ TO SOLVE B.B PROBLEMS.

$$\frac{1}{2p m_p^2}\varphi_{ini}^2 - \frac{p}{4} > 60$$

$$\varphi_{ini}^2 > \left(60 + \frac{p}{4}\right) 2p m_p^2$$

$$\varphi_{ini} > \sqrt{\left(60+\frac{p}{4}\right)2p}\; m_p$$

$p \simeq 1$: $\frac{p}{4} \ll 60$

$$\rightarrow \boxed{\varphi_{ini} \gtrsim \sqrt{120p}\, m_p} \quad \varphi_{ini} \gtrsim 10 m_p \text{ WHICH IS A WORRY.}$$

$\varphi_{EXIT} \simeq \sqrt{120p}\; m_p \rightarrow \simeq$ PIVOT SCALE EXITS.

NORMALISATION OF POWER SPECTRUM:

$$P_S^2 \simeq \frac{H^4}{\dot\varphi^2} = \frac{(3H^2)^3}{3(3H\dot\varphi)^2} = \frac{V^3}{3 m_p^6 V_{,\varphi}^2}$$

$$V_{,\varphi} \simeq p\mu^{4-p}\lambda\varphi^{p-1}$$

$$\frac{V^3}{V_{,\varphi}^2} = \frac{\mu^{4-p}\lambda\varphi^{p+2}}{p^2}$$

$$P_S^2 \simeq \frac{\mu^{4-p}\lambda\varphi^{p+2}}{3 m_p^6 p^2}$$

$\varphi_{EXIT} \simeq \sqrt{120 p\, m_p^2}$.

$$P_\zeta^2 \simeq \frac{\mu^{4-p}\lambda\,(120 p\, m_p^2)^{\frac{p+2}{2}}}{3 m_p^6 p^2} \simeq 10^{-9} \text{ (COBE)}.$$

$$\mu^{4-p}\lambda \simeq \frac{3 m_p^6 p^2 \cdot 10^{-9}}{(120 p\, m_p^2)^{\frac{p+2}{2}}}$$

$p = 2$: $\quad V(\varphi) = \mu^2 \lambda \varphi^2 = \frac{1}{2} m^2 \varphi^2$

$$\rightarrow \quad \frac{1}{2} m^2 \simeq \frac{3 m_p^6 \cdot 4 \cdot 10^{-9}}{(120 \cdot 2 \cdot m_p^2)^2}$$

$$\sim \frac{10^{-8}}{10^4}\, m_p^2$$

$$\sim 10^{-12}\, m_p^2$$

$m \sim 10^{-6}\, m_p$. $\rightarrow$ EVEN THOUGH $\varphi_{INI} \gtrsim 10 m_p$, UNDER GOOD THEORETICAL CONTROL IN TERMS OF $V \ll m_p^4$.

~~[illegible]~~

4 b) Radiative stability : $\mu^{4-p} \lambda \phi^p$

$p = 2 \rightarrow$ radiatively stable

~~$1 < p < 2$:~~

For $p < 2$, there is a term of $\sim \phi^2$ allowed of the form $k\phi^2$, or ϕ^2 / M_{eff}^2.

This term has the ability to drive the scale to M_{eff}, as opposed to μ.

Thus, the $\mu^{4-p} \lambda \phi^p$ term is not radiatively stable, similar to the standard model Lagrangian.

c) Field range is limited in order to make it such that things are still compactifiable.

For smaller p the range gets restricted even further