

Making predictions in the multiverse

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ABSTRACT: I describe reasons to think we are living in an eternally inflating multiverse where the observable “constants” of nature vary from place to place. The major obstacle to making predictions in this context is that we must regulate the infinities of eternal inflation. I review a number of proposed regulators, or measures. Recent work has ruled out a number of measures by showing that they conflict with observation, and focused attention on a few proposals. Further, several different measures have been shown to be equivalent. I describe some of the many nontrivial tests these measures will face as we learn more from theory, experiment, and observation.

Contents

1. Introduction	1
2. Is eternal inflation in our past?	4
3. Predictions in Eternal Inflation	5
3.1 Local Measures	6
3.2 Global Measures	8
4. Towards deriving a cutoff	11
4.1 The new lightcone time cutoff	11
5. Equivalences between measures; global-local duality	14
6. Tests of measures and the landscape	15
6.1 Particle Physics	15
6.2 Predicting the cosmological constant	15
6.3 The Boltzmann Brain problem	16
6.4 Alarming implications of geometric cutoffs: The end of time	17
7. Summary: the status of measures	18

1. Introduction

Weinberg’s successful prediction of the cosmological constant [1] suggests that we are living in a very large universe where the “constants” of nature vary from place to place. In the years since Weinberg’s prediction, advances in string theory have lent theoretical support to this idea. While string theory is a unique 11-dimensional theory with no free parameters, it seems to contain a huge landscape of solutions with 4 large dimensions and 7 tiny dimensions [2]. At the energy scales we can access, these different solutions look like distinct 4-dimensional theories with different values for the physical constants, such as the cosmological constant Λ and the Higgs mass.

Furthermore, eternal inflation occurs naturally in the string landscape. Starting from finite initial conditions, eternal inflation produces an arbitrarily large spacetime volume in the inflating false vacuum. Inflation ends locally, producing “pocket universes” where the fields settle into one of the vacua. Globally, inflation never ends, and all of the vacua of string theory are produced as pocket universes within a vast eternally inflating cosmology often called the *multiverse*.

If the fundamental theory produces such a large universe where the low-energy laws of physics vary from place to place, how are we to make predictions? First, it is natural to use the anthropic principle. In this context the anthropic principle is simply a controversial name for a mundane selection effect: when we are predicting the results of an observation, we can focus on the parts of the multiverse where observations can occur. More precisely, if we had complete knowledge of the multiverse, the relative probability of two different observations is given by *counting* the number of observations of each type,

$$\frac{p_A}{p_B} = \frac{N_A}{N_B}. \quad (1.1)$$

For example, to predict the value of Λ we count the number of observations of different values in the multiverse.

In this setting, the constants we observe may be fine-tuned from the conventional point of view. For example, Λ is 123 orders of magnitude smaller than its natural value. However, we expect our observations will not appear fine-tuned among regions where observations occur. Weinberg’s prediction that Λ would be detected soon was based on this assumption.

Because computing the cosmological constant requires a theory of quantum gravity, one could hope that the observed value of Λ will turn out to be conventionally natural, without appealing to anthropic selection in the multiverse. However, there are a number of other parameters such as the charge of the electron that appear fine-tuned for life, providing additional evidence for the multiverse.

It is disappointing that the fundamental theory gives only a probabilistic prediction for quantities like Λ , because we will never be able to measure the value of Λ in other parts of the multiverse. But our disappointment does not mean that the theory is wrong. In fact, we already accept that a fundamental theory may give only a probabilistic prediction for a quantity that we can only measure once: for example, the our best theory of the early universe gives only a statistical prediction for the $\ell = 3$ modes in the CMB. But no one questions the moral integrity of the theory of slow roll inflation as a result. What is needed for the theory of the multiverse to take its place as a well-accepted physical theory is simply better theoretical control and more opportunities to compare with experiment.

The major obstacle of principle to implementing the program of making predictions by counting observations in the multiverse is the existence of divergences. Eternal inflation produces not just a very large universe, but an infinite universe containing an infinite number of pocket universes, each of which is itself infinite. Therefore both the numerator and the denominator of (1.1) are infinite. We can define the ratio by regulating the infinite volume, but it turns out that the result is highly regulator-dependent.

There are two possible conclusions: either the proposal (1.1) is fundamentally ill-defined, or quantum gravity gives a way of defining it. It would seem that since string theory is a consistent theory of quantum gravity, it should be able to answer if and how equation (1.1) is defined. Unfortunately, this question is not yet tractable. We have an exact nonperturbative description of spacetimes with $\Lambda < 0$ that are asymptotically Anti-de Sitter (AdS) in terms of a dual conformal field theory, the famous AdS/CFT correspondence. Similarly, Matrix theory is a nonperturbative description of spacetimes with $\Lambda = 0$ and Minkowski asymptotics. But we do not have the corresponding description of spacetimes with $\Lambda > 0$ and eternally inflating asymptotics; even more generally, we do not have a rigorous description of *any* cosmology. One reason for this difficulty is that the asymptotic behavior of eternal inflation- more and more pocket universes in the future- is much more complicated than the asymptotic behavior of AdS spacetimes, which do not fluctuate near the boundary.

Because we lack the tools to address eternal inflation in a completely rigorous way within string theory, our understanding necessarily relies on approximations. It would be extremely interesting to develop the necessary tools to conclusively establish the existence or absence of the multiverse. An alternative is to take a more phenomenological approach, trying to understand some predictions of the theory before it is completely worked out. I will mention some hints that quantum gravity *does* regulate equation (1.1) in section 3. This is fortunate because I am not aware of any other proposal for how to make predictions in the context of the landscape.

The first step in making any prediction in eternal inflation is to regulate the infinities to make (1.1) well-defined. A procedure for regulating the infinities is called a *measure*. I will focus on *geometric cutoffs*: measures that supplement the semiclassical treatment of eternal inflation with a prescription for cutting off the infinite spacetime volume. In section 3 I will describe a number of simple measure proposals and their properties. Several of these proposals preserve a property that makes eternal inflation particularly attractive for making predictions: the late-time behavior is independent of the initial conditions, so all we need to know about the initial conditions is that they allow eternal inflation to occur. The late-time attractor behavior, however, does depend on the choice of cutoff.

In section 4, I will describe some steps towards deriving the measure from quantum gravity. Fortunately, as I will discuss in section 5, we have discovered surprising equivalences between proposals that sound very different. Furthermore, some of the reasonable proposals conflict strongly with observation and can be ruled out. This finally leaves us with only two or three distinct proposals.

Then I describe, in section 6, the many future tests that proposals will have to pass, emphasizing that all extant proposals could easily be ruled out in the near future. Finally I conclude with a brief summary of the status of measure proposals.

As a prelude, in section 2 I describe a simple set of assumptions leading to the conclusion that eternal inflation occurs in our past.

This is a personal review of the state of the field, reflecting my own prejudices and ignorance. In particular, I have not made an effort to cite every relevant paper on the subject. Where I have included citations I have tried to refer to useful references rather than original work. This may irritate my friends who work on eternal inflation, but it will hopefully lead to a more readable article.

2. Is eternal inflation in our past?

It sounds contradictory to ask whether eternal inflation is in our past, since eternal processes never end. But in the theory of eternal inflation, some regions of spacetime do stop inflating. The question is whether a long period of eternal inflation occurred before our pocket universe formed.

In this section I will spell out a set of assumptions that leads to the conclusion that eternal inflation is in our past. While many readers will find this a boring exercise, some physicists believe the conclusion is obviously wrong. Therefore, I think it is worth stating the assumptions clearly. Those who are already convinced that eternal inflation is in our past can skip this section.

Assumption 1: The potential allows eternal inflation to occur. For simplicity, we focus here on eternal inflation that occurs in a metastable false vacuum. In order for the landscape to allow eternal inflation to occur, it must contain at least one false vacuum whose decay rate is slower than its Hubble expansion rate,

$$\Gamma \lesssim H^4 . \tag{2.1}$$

Because the decay of a metastable vacuum is a nonperturbative process, Γ is naturally exponentially small. Further, string theory seems to contain a very large number of metastable false vacua. It would take a vast conspiracy to avoid having at least one vacuum that satisfies the bound above.

Assumption 2: Initial conditions. Suppose the theory contains a false vacuum whose decay rate is slow enough to satisfy (2.1). For eternal inflation to get started, we need to begin with several Hubble volumes that are (a) in the false vacuum and (b) are dominated by vacuum energy. The required initial conditions are generic in the technical sense: arbitrary small perturbations of the initial conditions will still allow eternal inflation to occur. Therefore the initial conditions that allow for eternal inflation form an open set in the set of all initial conditions.

Of course in a sense the initial conditions allowing for eternal inflation are very special. We do not know the correct theory of initial conditions, so it is hard to say how special they are. What I will assume is that the theory of initial conditions gives a nonzero probability to begin in the open set of initial conditions that allows for eternal inflation to begin.

I also assume that the initial conditions are spatially finite. If the initial conditions are defined on an infinite spatial slice, we already have a problem of infinities before even considering the dynamics of eternal inflation.

Assumption 3: Typicality. In determining where in the multiverse we are living, we make the assumption of typicality: we are equally likely to be anywhere consistent with our data. This is called the “principle of indifference.”

With our assumptions, there is a finite probability for eternal inflation, which results in an infinite number of observations, so we can ignore any finite number of observations.¹ Then to make predictions we can focus on the eternally inflating branch of the wave function. Within this branch, again we can ignore the finite number of observations that occur at early times. Thus with these assumptions a long period of eternal inflation is in our past.

3. Predictions in Eternal Inflation

Having argued that we are living during the late time era of eternal inflation, what are the predictions? If inflation were not quite eternal, but just led to an extremely large spacetime, the natural way to make predictions would be to count the number of events

¹This conclusion relies on an assumption about how to implement the typicality assumption when there is a probability distribution over how many observations occur [4]. I advocate first constructing the ensemble of probabilities and then using typicality within that wider ensemble. Page calls this choice *observational averaging*. As a simple example, suppose our theory is that God flips a fair coin, and if it is heads he makes one earth, while if it is tails he makes two earths that are far apart. Suppose we are about to do some observations that will determine whether there is another earth out there. I conclude the probability of observing another earth is 2/3. This turns out to be a controversial conclusion among philosophers; it is one version of the “sleeping beauty paradox.”

of different types, as in equation (1.1). However, this prescription becomes ambiguous if inflation is truly eternal.

One possibility at this point is to conclude that the ratios we want to compute are just not gauge invariant, and we are thinking about the problem wrong. From the point of view of semiclassical gravity this is the obvious conclusion because no principle within the theory gives a preferred way of defining (1.1).

However, there are some hints that in quantum gravity the ratio N_A/N_B may be well-defined. The infinities causing the ratio to be ill-defined come from counting events in causally disconnected regions of spacetime. We have learned from studying black holes that attempting to use semiclassical quantum gravity in causally disconnected regions of spacetime can lead to confusion. In the case of black holes, semiclassical analysis led to the conclusion that black holes destroy information. Even though the analysis seemed to be in a regime of low curvatures where semiclassical gravity is a good approximation, we now know that in a full theory of quantum gravity evolution is unitary.

The lesson I and many others take away from black hole physics is that semiclassical gravity can be trusted only within a single *causal diamond*. For a given worldline, the causal diamond is the region of spacetime that can send signals to and receive signals from the worldline; it is the largest region that can be probed in principle by a single observer.

In de Sitter space, the exponential expansion causes spatially separated points to fall out of causal contact with each other. The infinities of eternal inflation arise from these spacetime regions that are out of causal contact with each other, so the analogy with black holes suggests that the infinities are figments of our semiclassical imaginations². Given this encouragement and a dearth of other proposals, we will pursue the idea that eternal inflation is the right machine for making predictions from the string theory landscape.

3.1 Local Measures

The causal diamond measure. The causal diamond cutoff of Bousso [5] is motivated by the lessons of black hole physics. This cutoff keeps only those events occurring within a single causal diamond.

One still must specify the initial conditions for the diamond. The simplest option is to say that the specification of initial conditions is a separate problem from the measure problem. Another possibility is to define a rule for going from the global picture of the eternally inflating spacetime to an ensemble of causal diamonds.

²An exception is the causal patch of a worldline that enters a Minkowski vacuum, which can contain an infinite number of observations. I will return to this example later.

The most basic question is whether the causal diamond cutoff succeeds in regulating the infinities. The answer is yes, as long as the proper time along the central worldline is finite; then the volume of the causal diamond will be finite. However, for any reasonable choice of initial conditions there is a nonzero probability for a worldline to tunnel to a supersymmetric $\Lambda = 0$ bubble. Once inside the $\Lambda = 0$ bubble, it is believed that there is a nonzero probability for the worldline to attain infinite proper time [6]. These infinite worldlines lead to divergences, as Bousso already realized in his original paper, so the measure is defined to count only worldlines of finite length.

The causal diamond cutoff regulates the infinities of eternal inflation so strictly that the late-time attractor behavior disappears as well. The number of events of different types computed according to the causal diamond cutoff depends on the choice of initial conditions. So this cutoff is NOT a prescription for understanding the attractor behavior of eternal inflation. Instead, it tells us that the attractor behavior is an artifact of the same global picture that gave us infinity in the first place.

There are two simple variations on the causal diamond cutoff that are not quite as well motivated from black hole physics:

- **The apparent horizon measure.** This cutoff includes those events within the apparent horizon of the central geodesic [8], rather than keeping the entire causal diamond. This measure is very similar to the causal diamond measure and has not received as much attention, so I will not discuss it further here.
- **The fat geodesic measure.** Finally, instead of keeping all events within the causal diamond, one can only keep those events within a fixed physical volume centered on the geodesic [15]. This proposal was motivated by an equivalence to a global cutoff, and we will discuss it more in the next section.

The census taker cutoff. An alternative to throwing out the causal diamonds that become infinitely large is to focus on them. The “census taker cutoff” is the most famous unpublished cutoff prescription [7]. Consider a worldline that ends in a hat, and therefore has infinite length. Counting everything within the causal diamond of this worldline is less infinite than the entire global multiverse, but it is still infinite. However, suppose we just count all events that can send a signal to the central worldline before proper time τ . This is a finite set. We can now take the limit $\tau \rightarrow \infty$.

It is plausible that the probabilities defined in this way are independent of initial conditions. This prescription has the advantage of keeping the attractor behavior of eternal inflation while restricting to a single causally connected region. As far as anyone knows, the census taker cutoff is not ruled out by observation.

However, there is one major aesthetic problem that accounts for the fact that it has not been published. Imagine that we are trying to send a signal to the census taker. We need to arrange to live in a region of spacetime near the $\Lambda = 0$ bubble. We want to send a signal through the domain wall that separates us from the $\Lambda = 0$ bubble.

It turns out that our ability to be counted in the census depends crucially on the behavior of the domain wall between our bubble and the $\Lambda = 0$ bubble. Because the $\Lambda = 0$ bubble has smaller vacuum energy, we know that a free falling observer inside the $\Lambda = 0$ bubble will see the domain wall accelerate away from him. However, it can still accelerate towards us or away from us; it is a counterintuitive feature of general relativity that the domain wall can appear to accelerate away as seen from both sides. If the domain wall tension is low enough, it will accelerate towards us. In this case it is not difficult to be counted, because the domain wall could well come into our future lightcone.

However, if the tension is too big, then the domain wall will accelerate away from us, and the region of our vacuum that can send signals to the census taker will have only a microscopic thickness. For our value of the cosmological constant, the critical tension that divides these two behaviors is

$$T \sim \frac{\sqrt{\Lambda}}{G_N} \sim (\text{GeV})^3 \quad (3.1)$$

It seems absurd that whether we are counted in the census depends on such details as the tension of a particular domain wall. To put it another way, the census taker cutoff would predict that the domain wall between our vacuum and a supersymmetric $\Lambda = 0$ vacuum is very likely to have a small tension. This dependence on arcane details makes the census taker cutoff aesthetically unattractive.

The census taker description of eternal inflation [9] still may be a valuable one, but probably not in the simple sense of only counting events that take place within the census taker's backward lightcone.

3.2 Global Measures

Historically, the local measures were only developed long after the global time cutoffs. The idea of a global time cutoff is to pick some preferred global time variable in the multiverse. We first count only events that happen before some cutoff time t_0 , then take the limit $t_0 \rightarrow \infty$. Assuming the initial conditions are finite, there is only a finite spacetime volume before a finite time t_0 , so this procedure succeeds in regulating the infinities; in all known cases the limit $t \rightarrow \infty$ is well-behaved. Several of these cutoffs were introduced by Linde and collaborators in 1993 [10], although the precise procedure for making predictions was only clarified later.

The proper time cutoff. Perhaps the simplest definition is to use the proper time as a cutoff [10]. Begin with a finite spacelike initial surface Σ_0 that we will define to be $\tau = 0$, and erect a congruence of timelike geodesics orthogonal to that surface. The time at some other point is given by the proper time from the initial surface, measured along the geodesic in the congruence that connects the given point to the initial surface. We first count all events before some proper time τ_0 , then take the limit $\tau_0 \rightarrow \infty$. The resulting probabilities will be independent of the initial conditions.

There are various possible technical issues with this definition; for example, we have to decide how to define the time if there are two geodesics connecting the given point to the initial surface, as will happen to the future of caustics. These issues are not worth worrying about because this proposal has more severe problems: namely, it suffers from the “younghness problem” [11, 12]. This cutoff predicts that we are incredibly unlikely to live at such a late time, 13.7 billion years after our local big bang. The probability to live only, say, 13 billion years after the big bang is larger by the enormous factor $\exp(10^{60})$ [12]. Therefore, the proper time cutoff conflicts with observation.

Linde and collaborators have attempted to modify the proper time cutoff to resolve this conflict with observation [13]. I personally have not been able to understand from their work a relatively simple, well-defined modification that brings the cutoff into agreement with observation.

The scale factor time cutoff. The scale factor cutoff is defined similarly to the proper time cutoff, by considering the same geodesic congruence orthogonal to an initial surface Σ_0 . Now, however, the time is measured by the expansion rather than by the proper time. Along a geodesic, the scale factor time is defined by

$$dt = Hd\tau \tag{3.2}$$

where τ is the proper time and H is the local expansion of the geodesic congruence. Roughly, this means that time advances by one e-folding everywhere. This cutoff first appeared (as far as I know) in the work of Linde and collaborators in 1993 [10], and was first defined carefully by De Simone, Guth, Salem, and Vilenkin in 2008 [14].

Again, the definition (3.2) brings up various technical questions. In this case I will mention them because they are the worst aspect of this cutoff procedure. The main issue is that the above definition becomes ambiguous in the future of caustics. In the future of a caustic, defining the time by the above equation is not unique because there is more than one geodesic leading to the point under consideration. [14] made a choice for what to do, but a result is that in order to compute the current scale factor time we need to understand the intricacies of geodesic motion around our galaxy [15]. The proposal does not conflict with observation as far as we know, but it just seems wrong

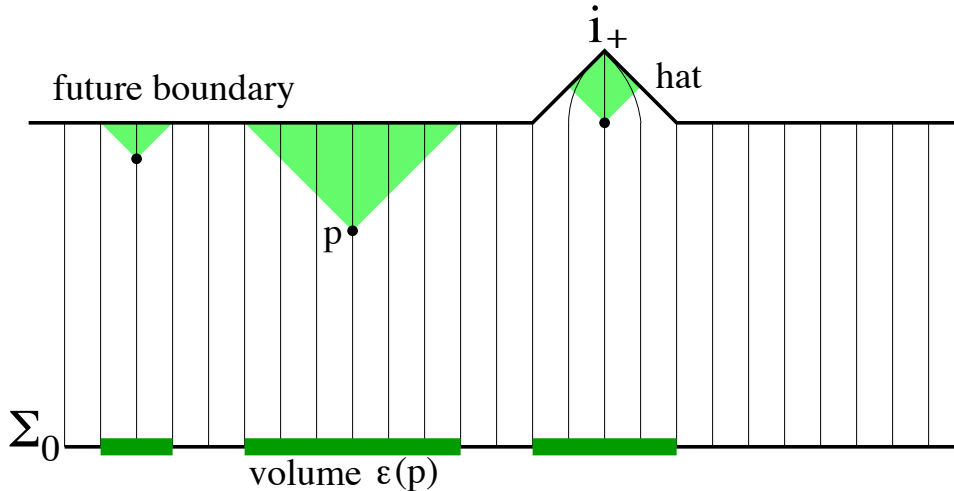


Figure 1: To compute the lightcone time of an event, construct its future lightcone and project back to the initial surface Σ_0 along the geodesic congruence. The resulting volume on Σ_0 gives the lightcone time of the event. Figure courtesy of Raphael Bousso.

that the predictions should depend on the gory details of the motion of geodesics in our galaxy. I will return briefly in section 4 to how the proposal might be modified to avoid this issue.

The lightcone time cutoff. The lightcone time cutoff makes use of the same geodesic congruence described above. To find the time at some spacetime point, construct its future lightcone. The future lightcone will capture some of the geodesics, as shown in figure 3.2. Following these captured geodesics back to the initial surface, find the volume ϵ of the captured geodesics on the initial surface. The lightcone time is related to the volume by

$$t \equiv -\frac{1}{3} \log \epsilon . \quad (3.3)$$

This time variable was first defined by Vilenkin and collaborators [16] in the more restricted context of counting the number of bubbles of each type. In that context it is extremely natural to consider the geodesics in the future lightcone because bubbles expand out from the nucleation point at the speed of light. It was first proposed as a cutoff- that is, as a rule for counting any type of event- by Bousso [17].

There is a sense in which the lightcone time is better defined than the scale factor time: no ambiguity arises in computing the volume when the geodesic congruence has caustics. Similarly, the lightcone time of an event depends only weakly on details of geodesic motion since most of the geodesics in the future lightcone never enter galaxies. (See however [18] for possibly significant effects of structure formation.)

On the other hand, Vilenkin has complained that lightcone time suffers from a problem he calls “shadows of the future”: the lightcone time of an event depends on what happens in the future of that event, because future events can affect the size of the lightcone.

4. Towards deriving a cutoff

We would prefer to be able to derive the cutoff directly from theory. An obstacle to this is that the rigorous description of eternal inflation in string theory is not known. But we do have some idea for what a rigorous description might look like, and even without fully understanding the theory we can make some progress towards deriving a cutoff prescription.

We do not believe that completely stable de Sitter space is possible in string theory. If it were, it would be natural to assume that there is a dual, nongravitational theory living on the conformal boundary of the spacetime [20]. Looking at holographic entropy bounds [19] informs us that future infinity is a holographic screen for de Sitter spacetime, and thus the natural place for the dual theory.

The fact that the dS/CFT correspondence has not been extremely successful is probably due to the problem that completely stable de Sitter space does not seem to exist in string theory. However, a dual description on future infinity still seems natural for the part of the spacetime that is still in the false vacuum.

The main theoretical problem has to do with vacua with $\Lambda \leq 0$. For bubbles with $\Lambda \leq 0$, it is not natural to expect that the bulk physics is captured by a dual description that lives on future infinity. This can be seen from the conformal diagrams in figure 2. It seems that these bubbles make holes in the theory on future infinity. This is of course not something that can happen in a conventional field theory; it suggests that the metric of the boundary must fluctuate. There were already hints of this perturbatively: de Sitter space inevitably has gravitational fluctuations that freeze out on scales larger than the horizon, so the natural state in the bulk corresponds to a sum over boundary geometries.

Given these vague outlines of a dual theory, we can try to derive a cutoff. The basic idea is the UV/IR correspondence familiar from AdS/CFT: in this case a short-distance (UV) cutoff in the boundary theory corresponds to a late-time (IR) cutoff in the bulk. This late-time cutoff renders the spacetime volume finite and therefore regulates the infinities.

4.1 The new lightcone time cutoff

The first step is to define a UV cutoff in the boundary theory. In order to how to

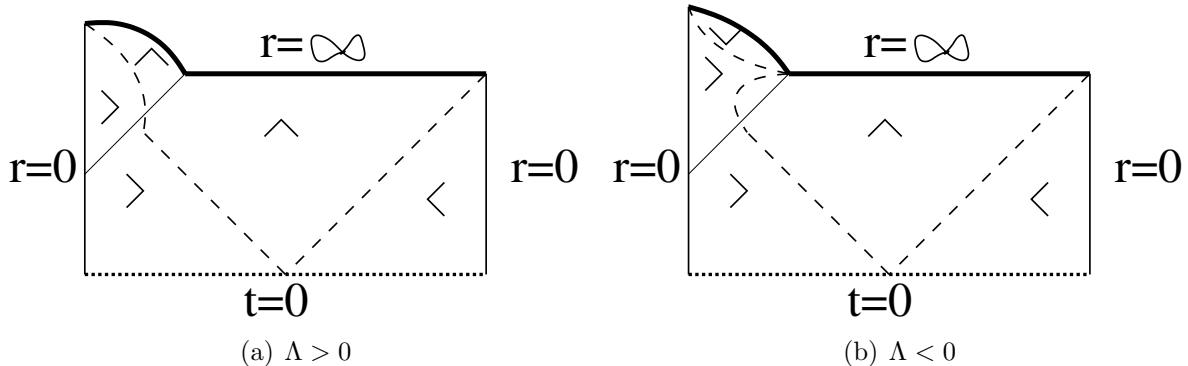


Figure 2: Conformal diagrams for bubbles with positive and negative cosmological constant. The Bousso wedges indicate the null directions in which the sizes of spheres are increasing. The tips of the wedges point in the direction of increase. Entropy bounds dictate that if we think of the wedges as arrows, they point towards the natural holographic screens [19]. One can see that future infinity is a natural holographic screen for the $\Lambda > 0$ bubble (left), but not for the $\Lambda < 0$ bubble.

regulate the boundary theory in the UV, we first need to know the metric on the boundary. In the physical spacetime, the metric diverges at future infinity. To define the metric on future infinity, we do a conformal transformation of the bulk spacetime. However, there is no unique way to choose the “right” conformal transformation. The result is that, for a given physical spacetime, the boundary metric is only defined up to conformal transformations,

$$\tilde{g}_{ab} = e^{2\phi} g_{ab} \quad (4.1)$$

where ϕ is an arbitrary function. This is a problem, because in order to put in a uniform UV cutoff in the boundary theory, we need to know the metric. There is no fiducial choice, and different natural choices will correspond to physically different cutoff prescriptions.

At this point it would be nice to know more about the boundary theory. But in our ignorant state, we can hypothesize a simple, natural way to fix the conformal factor of the boundary metric: we demand [22]

$${}^{(3)}R = \text{const.} \quad {}^{(3)}V = 1 . \quad (4.2)$$

Some nontrivial mathematics guarantees that this choice can be made and generically completely fixes the ambiguity.

This choice of metric is less arbitrary than it seems at first. First of all, as long as we demand the scalar curvature remain finite, we would get the same physical predictions. Second, Vilenkin has argued recently [21] that the only sensible conditions for fixing the metric are functions of the scalar curvature and its derivatives.

The bulk-boundary mapping. Having defined a UV cutoff in the boundary theory, we now need to know how to relate this to an IR cutoff in the bulk. Even in the better-understood case of AdS/CFT, it is highly nontrivial to work out how a given UV cutoff in the boundary theory translates into the bulk. I will continue to focus on bulk cutoffs that are completely geometric: a sharp cutoff surface bounds the events that are counted. Any simple cutoff in the boundary theory will probably not correspond to such a simple bulk cutoff- instead, the description of bulk events will grow fuzzy as they approach the cutoff.

One simple recipe is the following: given a bulk spacetime point, construct its future lightcone. Find the volume of the region on the future boundary that is contained inside the future lightcone. Keep only those bulk points whose future lightcones have volumes bigger than some minimum size. This can be stated by defining a time variable

$$t \equiv -\frac{1}{3} \log V \tag{4.3}$$

where V is the volume on future infinity that is inside the future lightcone. The new lightcone time is clearly very similar to the old lightcone time. The only difference is that in the old lightcone time the volume is measured on the initial surface Σ_0 , while in the new lightcone time it is measured on future infinity.

Having defined a time variable- the “new lightcone time”- we can use this as a regulator as before. We will return in the next section to further properties of this measure.

It is also possible to consider other ways of mapping a UV cutoff on the boundary to an IR cutoff in the bulk. Instead of using the future lightcone, Garriga and Vilenkin [33] consider trying to describe a bulk process with minimum resolution λ_{\min} . The boundary size is given by propagating the minimal length along geodesics up to future infinity. Then we only count events whose boundary size is bigger than the UV cutoff. It is not completely clear how to choose the scale λ_{\min} in general. For example, if we want to count observations of the cosmological constant, what minimum resolution should we demand? On the other hand, this proposal encodes a property one would intuitively expect: It seems reasonable that ants will cease to be resolved in the cutoff theory before people due to their smaller size and mass.

More recently, Vilenkin has argued that the most natural bulk/boundary mapping leads to a cutoff on bulk surfaces with constant comoving apparent horizon [21].

We should not be surprised that different authors have come to different conclusions about the best way to perform the bulk/boundary mapping. Even in the much better understood case of AdS/CFT, the bulk/boundary map is only simple on length scales large compared to the curvature radius. Some progress has been made in describing

smaller objects in AdS (see, for example, [24]), but the UV/IR relation becomes much more complicated. In particular, a simple UV cutoff in the boundary theory probably does not correspond to a sharp IR cutoff on the geometry in the bulk except on scales larger than the curvature radius; see [25] for recent progress in addressing this question.

The key point is that how exactly the bulk spacetime is cut off in the IR depends on the *details* of the UV cutoff in the boundary theory. Different UV cutoffs will give different prescriptions for cutting off the bulk. If the boundary theory is a conventional field theory, then the only quantities that make sense are those that are independent of the details of the cutoff. So if the boundary theory were a conventional field theory, we would have to conclude that the bulk quantities we are computing do not make sense, because they depend on the details of the cutoff. Fortunately, as we have discussed, there is evidence that the boundary theory is not a conventional field theory. It may be a theory that has a built-in UV cutoff.

In order to really fulfill our dream of deriving a cutoff and bring our subject onto firm theoretical ground, we will have to make progress in understanding the boundary theory, or whatever the correct description of eternal inflation in quantum gravity is.

5. Equivalences between measures; global-local duality

It is annoying to have so many reasonable-sounding measure proposals. One encouraging fact is that many of these proposals turn out to be equivalent to each other. First, as the terminology suggests, the new lightcone time cutoff is equivalent to the old lightcone time cutoff in the approximation that the bubbles are homogeneous FRW universes [22].

A more surprising correspondence has been discovered between the local measures and the global measures [17, 26, 15], called global-local duality. Recall that the local measures (except the census taker, which we will not discuss further) depend on initial conditions. The statement is that the global measures are equivalent to local measures *if* the initial conditions for the local measure is given by the attractor behavior of the global measure. For the measures under consideration, this initial condition is extremely simple: the geodesic should start in the most stable vacuum with positive cosmological constant.

Given this choice of initial condition, the lightcone time cutoffs, which are defined globally, are equivalent to the causal diamond cutoff [17, 26]. It is very encouraging that the new lightcone time, which we motivated by the UV/IR correspondence, turns out to be equivalent to counting events within a single causal diamond.

Similarly, the scale factor cutoff is equivalent, in the approximation that the geodesic congruence never stops expanding, to the “fat geodesic” cutoff [15].

So all of the measures discussed above reduce to only two proposals that are still in agreement with observation: the lightcone time cutoff and the scale factor cutoff. (We could count the global and local versions of the apparent horizon cutoff as a third possibility, but since these proposals are rather new and not radically different I will ignore them for brevity.)

So it all boils down to this: consider a geodesic that begins in the most stable de Sitter vacuum. Keep either all events within its causal diamond (lightcone time/ causal diamond), or within a fixed physical volume orthogonal to the geodesic (scale factor time/ fat geodesic).

6. Tests of measures and the landscape

There are many opportunities for measure proposals to conflict with observation. As described above, the proper time measure is very natural theoretically, but makes a completely wrong prediction about the observed age of the universe. There are many additional tests, and I will only briefly describe a few. The existing measure proposals pass these tests, as far as we know given our current knowledge of the landscape. However, as we learn more both theoretically and experimentally, it could easily happen that all the measures I have described here will be ruled out.

6.1 Particle Physics

One area where our knowledge of the landscape has so far limited our ability to make predictions is particle physics. We would like to be able to predict the supersymmetry breaking scale, among other quantities. One could easily imagine a scenario where the landscape predicts that the SUSY breaking scale is very high, near the Planck scale, while the LHC reveals low-energy SUSY. This type of development has the potential to rule out the entire multiverse framework for making predictions, and the SUSY breaking scale is just one of many tests of this type. Unfortunately, serious technical progress is needed before we can extract these predictions from the landscape.

6.2 Predicting the cosmological constant

The most persuasive piece of evidence for the landscape is Weinberg's prediction of the cosmological constant. However, there is also an opportunity for measures to fail once we generalize Weinberg's analysis and try to predict the cosmological constant allowing more parameters to vary. We argued recently [8] that for positive cosmological constant both the scale factor and lightcone measures are very successful in predicting the observed value of Λ , addressing concerns that Weinberg's prediction is not robust when other parameters are allowed to vary.

However, both measures are in danger of predicting that we are much more likely to observe a negative cosmological constant than a positive one, a conclusion already reached by Salem [27] and by Bousso and Leichenauer [28] in specific cases.

6.3 The Boltzmann Brain problem

Despite the fanciful sounding name, the Boltzmann Brain problem is a serious problem that has ruled out measures in the past [29] and poses a threat for currently popular measures as we learn more about the landscape.

The issue is that there are two ways observers can form in the multiverse. The first is the traditional way: a pocket universe forms, which then undergoes slow roll inflation, reheating, and structure formation. This process produces a large universe filled with matter. The second way structure can form is by a vacuum fluctuation in de Sitter space. De Sitter space has a finite temperature, so starting in empty de Sitter space, a fluctuation with mass M occurs with a rate given by

$$\Gamma \sim H^{-1} \exp\left(-\frac{M}{T}\right) \quad (6.1)$$

where the de Sitter temperature is $T = H/(2\pi)$. These fluctuations violate the second law, and they produce a mass in a universe that is otherwise completely empty.

As a concrete example of such a fluctuation, let us compute the expected time to fluctuate the earth out of our de Sitter vacuum. Let us specify that we want to fluctuate the earth in exactly its current state. The time for such a crazy fluctuation is

$$t \sim (10^{10} \text{ years}) \times e^{10^{92}}. \quad (6.2)$$

This is an unimaginably long time; however, it is far shorter than the recurrence time of our vacuum

$$t_{\text{rec}} \sim (10^{10} \text{ years}) \times e^{10^{123}} \quad (6.3)$$

Therefore, if our vacuum lives for of order the recurrence time, the number of ‘‘Boltzmann Earths’’ that fluctuate out of the vacuum within one causal patch is enormous, $N_{BE} \sim \exp(10^{123})$. This is superexponentially more than the number of planets that form within one causal patch by traditional structure formation; the number of ‘‘ordinary earths’’ is $N_{OE} \sim 10^{22}$.

If we focus attention just on our vacuum, and assume we are typical, then we conclude that if the lifetime is of order the recurrence time, we should be living on a Boltzmann Earth. But this conflicts with observation, because observers on Boltzmann Earths are living in an otherwise empty universe.

In the multiverse, observers form both in the ordinary way (“ordinary observers”) and from fluctuations (“Boltzmann Brains”). In order to agree with observation, it is important that the Boltzmann Brains do not vastly outnumber the ordinary observers. For the measures under consideration, the Boltzmann Brains will dominate if *any* vacuum in the landscape has a decay rate that is slower than its rate for producing Boltzmann Brains [29, 15, 30]. That is, to agree with observation, every vacuum must satisfy

$$\Gamma_{\text{decay}} > \Gamma_{BB} \tag{6.4}$$

This is a highly nontrivial bound; for example, it demands that our vacuum decay far faster than the recurrence time. As far as we know, the landscape satisfies this nontrivial bound [31], but we could find out otherwise any day.

6.4 Alarming implications of geometric cutoffs: The end of time

There is a sense in which all geometric cutoffs of eternal inflation predict a novel type of catastrophe: we could run into the cutoff, and time would end. I will describe the physics of the situation, but in the end it is a matter of judgment whether one should conclude that all geometric cutoffs are unsatisfactory. Predicting that time could end sounds crazy, but it does not contradict observation if the probability of encountering the end of time is small.

The main issue is this: in all cutoff prescriptions, a finite fraction of the observers who are born before the cutoff run into the cutoff before they die. This is true even for cutoffs that involve taking a late time limit due to the exponential growth of the spacetime. An analogy that is mathematically precise is a population that grows exponentially until doomsday. A finite fraction of everyone who has ever lived is alive on doomsday. This fraction does not go to zero as doomsday is taken later and later.

Because a finite fraction of the events happen close to the cutoff, we are forced to give some physical interpretation to observers who run into the cutoff. (If the population grew slower than exponentially, then as the cutoff was taken later and later the fraction of observers who run into the cutoff would go to zero, and we could forget about them.)

One can try to think of the cutoff as just a mathematical device for defining probabilities in an infinite set [33], and deny that the cutoff is a physical entity we could run into. But it is not so easy to escape the unpalatable consequences of the cutoff. The famous Guth-Vanchurin paradox [33, 32] illustrates this by showing that certain probabilities computed in the multiverse conflict with common sense expectations unless we take the end of time seriously as a catastrophe that could happen to us.

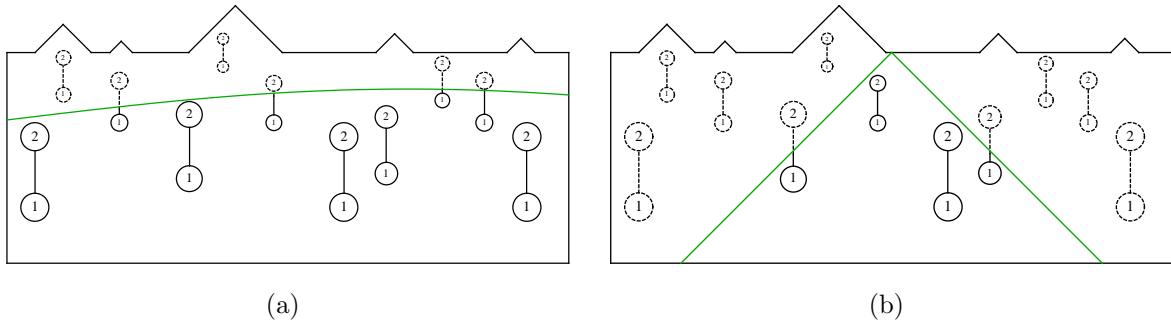


Figure 3: In both global (left) and local (right) cutoffs, a finite fraction of observers born before the cutoff run into the cutoff before they die. The fraction of observers who are cut off does not go to zero as the cutoff is taken later and later.

7. Summary: the status of measures

Our efforts over the last several years have focused attention on two simple measure proposals: the lightcone time cutoff and the scale factor time cutoff. These proposals can be described in a very simple way: follow a geodesic that begins in the most stable de Sitter vacuum in the landscape. The lightcone time cutoff counts only those events that are within the causal diamond centered on the geodesic, while the scale factor cutoff counts only those events within a fixed physical volume surrounding the geodesic.

Both proposals have passed a number of nontrivial tests, but may be ruled out in the near future as we learn more about the landscape.

There are two issues about these measures that concern me. The first is the end of time issue described above. While the measures agree with observation, predicting the end of time when there is no obvious physical mechanism seems wrong. On the other hand, avoiding the end of time conclusion seems to require a radical change in how we think about the measure problem.

There is also a more concrete issue: the fact that these measures do not seem to work well for negative cosmological constant. They have a strong tendency to predict we should observe $\Lambda < 0$, as I described in the previous section. Because of theoretical uncertainties, it is not yet clear that there is a strong conflict with observation, but there are clear hints.

Developing a more rigorous understanding of eternal inflation in string theory is crucial to putting this subject on firmer footing. We are beginning to have the vague outlines of a dual description for regions with $\Lambda > 0$, but we know very little about the correct description of regions with $\Lambda < 0$. The recent proposal of Maldacena [34] for a

dual description of crunches is very interesting, but it does not seem to describe well realistic cosmologies with a period of slow roll inflation.

The future is very exciting. We can look forward to attacking two of the biggest theoretical questions: the string theoretic description of cosmology and eternal inflation, and how to extract predictions from string theory. At the same time, our measures of the multiverse will continue to confront experiment and observation.

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