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Eternal inflation and its implications‡

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Abstract. I summarize the arguments that strongly suggest that our universe is the product of inflation. The mechanisms that lead to eternal inflation in both new and chaotic models are described. Although the infinity of pocket universes produced by eternal inflation are unobservable, it is argued that eternal inflation has real consequences in terms of the way that predictions are extracted from theoretical models. The ambiguities in defining probabilities in eternally inflating spacetimes are reviewed, with emphasis on the youngness paradox that results from a synchronous gauge regularization technique. Although inflation is generically eternal into the future, it is not eternal into the past: it can be proven under reasonable assumptions that the inflating region must be incomplete in past directions, so some physics other than inflation is needed to describe the past boundary of the inflating region.

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1. Introduction: the successes of inflation

Since the proposal of the inflationary model some 25 years ago [1–4], inflation has been remarkably successful in explaining many important qualitative and quantitative properties of the universe. In this article I will summarize the key successes, and then discuss a number of issues associated with the eternal nature of inflation. In my opinion, the evidence that our universe is the result of some form of inflation is very solid. Since the term *inflation* encompasses a wide range of detailed theories, it is hard to imagine any reasonable alternative. The basic arguments are as follows:

(i) *The universe is big*

First of all, we know that the universe is incredibly large: the visible part of the universe contains about 10^{90} particles. Since we have all grown up in a large universe, it is easy to take this fact for granted: of course the universe is big, it's the whole universe! In "standard" FRW cosmology, without inflation, one simply postulates that about 10^{90} or more particles were here from the start. However, in the context of present-day cosmology, many of us hope that even the creation of the universe can be described in scientific terms. Thus, we are led to at least think about a theory that might explain how the universe got to be so big. Whatever that theory is, it has to somehow explain the number of particles, 10^{90} or more. However, it is hard to imagine such a number arising from a calculation in which the input consists only of geometrical quantities, quantities associated with simple dynamics, and factors of 2 or π. The easiest way by far to get a huge number, with only modest numbers as input, is for the calculation to involve an exponential. The exponential expansion of inflation reduces the problem of explaining 10^{90} particles to the problem of explaining 60 or 70 e-foldings of inflation. In fact, it is easy to construct underlying particle theories that will give far more than 70 e-foldings of inflation. Inflationary cosmology therefore suggests that, even though the observed universe is incredibly large, it is only an infinitesimal fraction of the entire universe.

(ii) *The Hubble expansion*

The Hubble expansion is also easy to take for granted, since we have all known about it from our earliest readings in cosmology. In standard FRW cosmology, the Hubble expansion is part of the list of postulates that define the initial conditions. But inflation actually offers the possibility of explaining how the Hubble expansion began. The repulsive gravity associated with the false vacuum is just what Hubble ordered. It is exactly the kind of force needed to propel the universe into a pattern of motion in which any two particles are moving apart with a velocity proportional to their separation.

(iii) *Homogeneity and isotropy*

The degree of uniformity in the universe is startling. The intensity of the cosmic background radiation is the same in all directions, after it is corrected for the motion of the Earth, to the incredible precision of one part in 100,000. To get some feeling for how high this precision is, we can imagine a marble that is spherical to one part in 100,000. The surface of the marble would have to be shaped to an accuracy of about 1,000 angstroms, a quarter of the wavelength of light.

Although modern technology makes it possible to grind lenses to quarter-wavelength accuracy, we would nonetheless be shocked if we unearthed a stone, produced by natural processes, that was round to an accuracy of 1,000 angstroms. If we try to imagine that such a stone were found, I am sure that no one would accept an explanation of its origin which simply proposed that the stone started out perfectly round. Similarly, I do not think it makes sense to consider any theory of cosmogenesis that cannot offer some explanation of how the universe became so incredibly isotropic.

The cosmic background radiation was released about 300,000 years after the big bang, after the universe cooled enough so that the opaque plasma neutralized into a transparent gas. The cosmic background radiation photons have mostly been traveling on straight lines since then, so they provide an image of what the universe looked like at 300,000 years after the big bang. The observed uniformity of the radiation therefore implies that the observed universe had become uniform in temperature by that time. In standard FRW cosmology, a simple calculation shows that the uniformity could be established so quickly only if signals could propagate at 100 times the speed of light, a proposition clearly contradicting the known laws of physics. In inflationary cosmology, however, the uniformity is easily explained. The uniformity is created initially on microscopic scales, by normal thermal-equilibrium processes, and then inflation takes over and stretches the regions of uniformity to become large enough to encompass the observed universe.

(iv) *The flatness problem*

I find the flatness problem particularly impressive, because of the extraordinary numbers that it involves. The problem concerns the value of the ratio

$$\Omega_{\rm tot} \equiv \frac{\rho_{\rm tot}}{\rho_c} \ , \tag{1}$$

where $\rho_{\rm tot}$ is the average total mass density of the universe and $\rho_c = 3H^2/8\pi G$ is the critical density, the density that would make the universe spatially flat. (In the definition of "total mass density," I am including the vacuum energy $\rho_{\rm vac} = \Lambda/8\pi G$ associated with the cosmological constant Λ, if it is nonzero.)

By combining data from the Wilkinson Microwave Anisotropy Probe (WMAP), the Sloan Digital Sky Survey (SDSS), and observations of type Ia supernovae, the authors of Ref. [5] deduced that the present value of $\Omega_{\rm tot}$ is equal to one within a few percent ($\Omega_{\rm tot} = 1.012^{+0.018}_{-0.022}$). Although this value is very close to one, the really stringent constraint comes from extrapolating $\Omega_{\rm tot}$ to early times, since $\Omega_{\rm tot} = 1$ is an unstable equilibrium point of the standard model evolution. Thus, if $\Omega_{\rm tot}$ was ever *exactly* equal to one, it would remain exactly one forever. However, if $\Omega_{\rm tot}$ differed slightly from one in the early universe, that difference—whether positive or negative—would be amplified with time. In particular, it can be shown that $\Omega_{\rm tot} - 1$ grows as

$$\Omega_{\rm tot} - 1 \propto \begin{cases} t & \text{(during the radiation-dominated era)} \\ t^{2/3} & \text{(during the matter-dominated era)} \ . \end{cases} \tag{2}$$

Dicke and Peebles [6] pointed out that at $t = 1$ second, for example, when the processes of big bang nucleosynthesis were just beginning, $\Omega_{\rm tot}$ must have equaled one to an accuracy of one part in 10^{15}. Classical cosmology provides no explanation for this fact—it is simply assumed as part of the initial conditions. In the context of modern particle theory, where we try to push things all the way back to the Planck time, 10^{-43} s, the problem becomes even more extreme. If one specifies the value of $\Omega_{\rm tot}$ at the Planck time, it has to equal one to 59 decimal places in order to be in the allowed range today.

While this extraordinary flatness of the early universe has no explanation in classical FRW cosmology, it is a natural prediction for inflationary cosmology. During the inflationary period, instead of $\Omega_{\rm tot}$ being driven away from one as described by Eq. (2), $\Omega_{\rm tot}$ is driven towards one, with exponential swiftness:

$$\Omega_{\rm tot} - 1 \propto e^{-2H_{\rm inf}t} \ , \tag{3}$$

where $H_{\rm inf}$ is the Hubble parameter during inflation. Thus, as long as there is a long enough period of inflation, $\Omega_{\rm tot}$ can start at almost any value, and it will be driven to unity by the exponential expansion.

(v) *Absence of magnetic monopoles*

All grand unified theories predict that there should be, in the spectrum of possible particles, extremely massive particles carrying a net magnetic charge. By combining grand unified theories with classical cosmology without inflation, Preskill [7] found that magnetic monopoles would be produced so copiously that they would outweigh everything else in the universe by a factor of about 10^{12}. A mass density this large would cause the inferred age of the universe to drop to about 30,000 years! Inflation is certainly the simplest known mechanism to eliminate monopoles from the visible universe, even though they are still in the spectrum of possible particles. The monopoles are eliminated simply by arranging the parameters so that inflation takes place after (or during) monopole production, so the monopole density is diluted to a completely negligible level.

(vi) *Anisotropy of the cosmic background radiation*

The process of inflation smooths the universe essentially completely, but density fluctuations are generated as inflation ends by the quantum fluctuations of the inflaton field [8, 9]. Generically these are adiabatic Gaussian fluctuations with a nearly scale-invariant spectrum.

Until recently, astronomers were aware of several cosmological models that were consistent with the known data: an open universe, with $\Omega \cong 0.3$; an inflationary universe with considerable dark energy (Λ); an inflationary universe without Λ; and a universe in which the primordial perturbations arose from

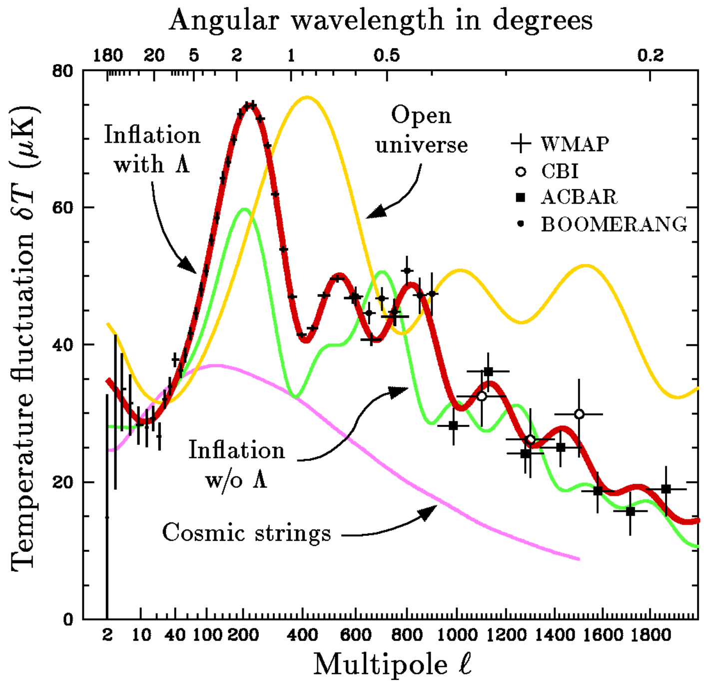

Figure 1. Comparison of the latest observational measurements of the temperature fluctuations in the CMB with several theoretical models, as described in the text. The temperature pattern on the sky is expanded in multipoles (i.e., spherical harmonics), and the intensity is plotted as a function of the multipole number ℓ. Roughly speaking, each multipole ℓ corresponds to ripples with an angular wavelength of $360°/\ell$.

topological defects such as cosmic strings. Each of these models leads to a distinctive pattern of resonant oscillations in the early universe, which can be probed today through its imprint on the CMB. As can be seen in Fig. 1 [20], three of the models are now definitively ruled out. The full class of inflationary models can make a variety of predictions, but the predictions of the simplest inflationary models with large Λ, shown on the graph, fit the data beautifully.

2. Eternal Inflation: Mechanisms

The remainder of this article will discuss eternal inflation—the questions that it can answer, and the questions that it raises. In this section I discuss the mechanisms that make eternal inflation possible, leaving the other issues for the following sections. I will discuss eternal inflation first in the context of new inflation, and then in the context of chaotic inflation, where it is more subtle.

2.1. Eternal New Inflation

The eternal nature of new inflation was first discovered by Steinhardt [24], and later that year Vilenkin [25] showed that new inflationary models are generically eternal.

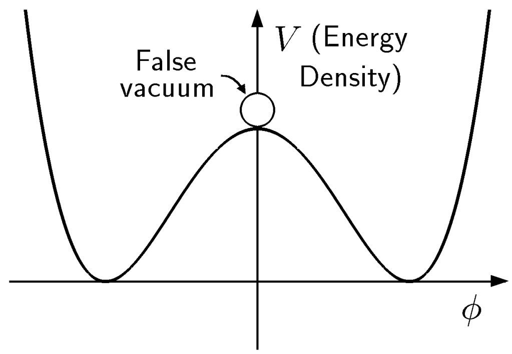

Figure 2. Evolution of the inflaton field during new inflation.

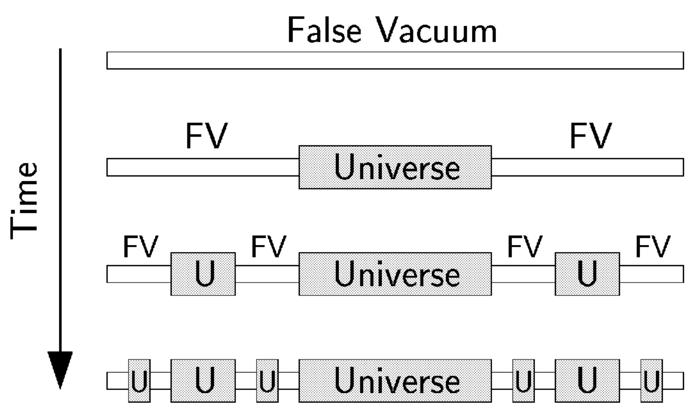

Figure 3. A schematic illustration of eternal inflation.

Although the false vacuum is a metastable state, the decay of the false vacuum is an exponential process, very much like the decay of any radioactive or unstable substance. The probability of finding the inflaton field at the top of the plateau in its potential energy diagram, Fig. 2, does not fall sharply to zero, but instead trails off exponentially with time [26]. However, unlike a normal radioactive substance, the false vacuum exponentially expands at the same time that it decays. In fact, in any successful inflationary model the rate of exponential expansion is always much faster than the rate of exponential decay. Therefore, even though the false vacuum is decaying, it never disappears, and in fact the total volume of the false vacuum, once inflation starts, continues to grow exponentially with time, ad infinitum.

Fig. 3 shows a schematic diagram of an eternally inflating universe. The top bar indicates a region of false vacuum. The evolution of this region is shown by the successive bars moving downward, except that the expansion could not be shown while still fitting all the bars on the page. So the region is shown as having a fixed size in comoving coordinates, while the scale factor, which is not shown, increases from each bar to the next. As a concrete example, suppose that the scale factor for each bar is three times larger than for the previous bar. If we follow the region of false vacuum as it evolves from the situation shown in the top bar to the situation shown in the second bar, in about one

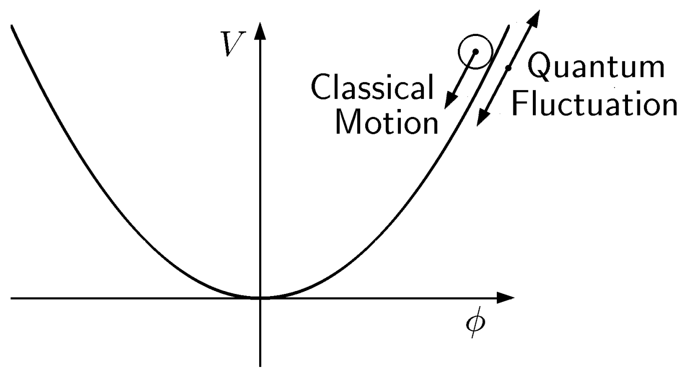

Figure 4. Evolution of the inflaton field during eternal chaotic inflation.

third of the region the scalar field rolls down the hill of the potential energy diagram, precipitating a local big bang that will evolve into something that will eventually appear to its inhabitants as a universe. This local big bang region is shown in gray and labelled "Universe." Meanwhile, however, the space has expanded so much that each of the two remaining regions of false vacuum is the same size as the starting region. Thus, if we follow the region for another time interval of the same duration, each of these regions of false vacuum will break up, with about one third of each evolving into a local universe, as shown on the third bar from the top. Now there are four remaining regions of false vacuum, and again each is as large as the starting region. This process will repeat itself literally forever, producing a kind of a fractal structure to the universe, resulting in an infinite number of the local universes shown in gray. There is no standard name for these local universes, but they are often called bubble universes. I prefer, however, to call them pocket universes, to avoid the suggestion that they are round. While bubbles formed in first-order phase transitions are round [27], the local universes formed in eternal new inflation are generally very irregular, as can be seen for example in the two-dimensional simulation by Vanchurin, Vilenkin, and Winitzki in Fig. 2 of Ref. [28].

The diagram in Fig. 3 is of course an idealization. The real universe is three dimensional, while the diagram illustrates a schematic one-dimensional universe. It is also important that the decay of the false vacuum is really a random process, while the diagram was constructed to show a very systematic decay, because it is easier to draw and to think about. When these inaccuracies are corrected, we are still left with a scenario in which inflation leads asymptotically to a fractal structure [29] in which the universe as a whole is populated by pocket universes on arbitrarily small comoving scales. Of course this fractal structure is entirely on distance scales much too large to be observed, so we cannot expect astronomers to see it. Nonetheless, one does have to think about the fractal structure if one wants to understand the very large scale structure of the spacetime produced by inflation.

Most important of all is the simple statement that once inflation happens, it produces not just one universe, but an infinite number of universes.

2.2. Eternal Chaotic Inflation:

The eternal nature of new inflation depends crucially on the scalar field lingering at the top of the plateau of Fig. 2. Since the potential function for chaotic inflation, Fig. 4, does not have a plateau, it is not obvious how eternal inflation can happen in this context. Nonetheless, Andrei Linde [30] showed in 1986 that chaotic inflation can also be eternal.

In this case inflation occurs as the scalar field rolls down a hill of the potential energy diagram, as in Fig. 4, starting high on the hill. As the field rolls down the hill, quantum fluctuations will be superimposed on top of the classical motion. The best way to think about this is to ask what happens during one time interval of duration $\Delta t = H^{-1}$ (one Hubble time), in a region of one Hubble volume H^{-3}. Suppose that ϕ_0 is the average value of ϕ in this region, at the start of the time interval. By the definition of a Hubble time, we know how much expansion is going to occur during the time interval: exactly a factor of e. (This is the only exact number in today's talk, so I wanted to emphasize the point.) That means the volume will expand by a factor of e^3. One of the deep truths that one learns by working on inflation is that e^3 is about equal to 20, so the volume will expand by a factor of 20. Since correlations typically extend over about a Hubble length, by the end of one Hubble time, the initial Hubble-sized region grows and breaks up into 20 independent Hubble-sized regions.

As the scalar field is classically rolling down the hill, the change in the field $\Delta\phi$ during the time interval Δt is going to be modified by quantum fluctuations $\Delta\phi_{\rm qu}$, which can drive the field upward or downward relative to the classical trajectory. For any one of the 20 regions at the end of the time interval, we can describe the change in ϕ during the interval by

$$\Delta\phi = \Delta\phi_{\rm cl} + \Delta\phi_{\rm qu} \; , \tag{4}$$

where $\Delta\phi_{\rm cl}$ is the classical value of $\Delta\phi$. In lowest order perturbation theory the fluctuations are calculated using free quantum field, which implies that $\Delta\phi_{\rm qu}$, the quantum fluctuation averaged over one of the 20 Hubble volumes at the end, will have a Gaussian probability distribution, with a width of order $H/2\pi$ [13, 31–33]. There is then always some probability that the sum of the two terms on the right-hand side will be positive — that the scalar field will fluctuate up and not down. As long as that probability is bigger than 1 in 20, then the number of inflating regions with $\phi \geq \phi_0$ will be larger at the end of the time interval Δt than it was at the beginning. This process will then go on forever, so inflation will never end.

Thus, the criterion for eternal inflation is that the probability for the scalar field to go up must be bigger than $1/e^3 \approx 1/20$. For a Gaussian probability distribution, this condition will be met provided that the standard deviation for $\Delta\phi_{\rm qu}$ is bigger than $0.61|\Delta\phi_{\rm cl}|$. Using $\Delta\phi_{\rm cl} \approx \dot{\phi}_{\rm cl} H^{-1}$, the criterion becomes

$$\Delta\phi_{\rm qu} \approx \frac{H}{2\pi} > 0.61\,|\dot{\phi}_{\rm cl}|\,H^{-1} \Longleftrightarrow \frac{H^2}{|\dot{\phi}_{\rm cl}|} > 3.8 \; . \tag{5}$$

We have not discussed the calculation of density perturbations in detail, but the condition (5) for eternal inflation is equivalent to the condition that $\delta\rho/\rho$ on ultra-long length scales is bigger than a number of order unity.

The probability that $\Delta\phi$ is positive tends to increase as one considers larger and larger values of ϕ, so sooner or later one reaches the point at which inflation becomes eternal. If one takes, for example, a scalar field with a potential

$$V(\phi) = \frac{1}{4}\lambda\phi^4 \ , \tag{6}$$

then the de Sitter space equation of motion in flat Robertson-Walker coordinates takes the form

$$\ddot{\phi} + 3H\dot{\phi} = -\lambda\phi^3 \ , \tag{7}$$

where spatial derivatives have been neglected. In the "slow-roll" approximation one also neglects the $\ddot{\phi}$ term, so $\dot{\phi} \approx -\lambda\phi^3/(3H)$, where the Hubble constant H is related to the energy density by

$$H^2 = \frac{8\pi}{3}G\rho = \frac{2\pi}{3}\frac{\lambda\phi^4}{M_p^2} \ . \tag{8}$$

Putting these relations together, one finds that the criterion for eternal inflation, Eq. (5), becomes

$$\phi > 0.75\,\lambda^{-1/6}\,M_p \ . \tag{9}$$

Since λ must be taken very small, on the order of 10^{-12}, for the density perturbations to have the right magnitude, this value for the field is generally well above the Planck scale. The corresponding energy density, however, is given by

$$V(\phi) = \frac{1}{4}\lambda\phi^4 = .079\lambda^{1/3}M_p^4 \ , \tag{10}$$

which is actually far below the Planck scale.

So for these reasons we think inflation is almost always eternal. I think the inevitability of eternal inflation in the context of new inflation is really unassailable — I do not see how it could possibly be avoided, assuming that the rolling of the scalar field off the top of the hill is slow enough to allow inflation to be successful. The argument in the case of chaotic inflation is less rigorous, but I still feel confident that it is essentially correct. For eternal inflation to set in, all one needs is that the probability for the field to increase in a given Hubble-sized volume during a Hubble time interval is larger than 1/20.

Thus, once inflation happens, it produces not just one universe, but an infinite number of universes.

3. Implications for the Landscape of String Theory

Until recently, the idea of eternal inflation was viewed by most physicists as an oddity, of interest only to a small subset of cosmologists who were afraid to deal with concepts that make real contact with observation. The role of eternal inflation in scientific thinking, however, was greatly boosted by the realization that string theory has no preferred vacuum, but instead has perhaps 10^{1000} [34, 35] metastable vacuum-like states. Eternal inflation then has potentially a direct impact on fundamental physics, since it can provide a mechanism to populate the landscape of string vacua. While all of these vacua are described by the same fundamental string theory, the apparent laws of physics at low energies could differ dramatically from one vacuum to another. In particular, the value of the cosmological constant (e.g., the vacuum energy density) would be expected to have different values for different vacua.

The combination of the string landscape with eternal inflation has in turn led to a markedly increased interest in anthropic reasoning, since we now have a respectable set of theoretical ideas that provide a setting for such reasoning. To many physicists, the new setting for anthropic reasoning is a welcome opportunity: in the multiverse, life will evolve only in very rare regions where the local laws of physics just happen to have the properties needed for life, giving a simple explanation for why the observed universe appears to have just the right properties for the evolution of life. The incredibly small value of the cosmological constant is a telling example of a feature that seems to be needed for life, but for which an explanation from fundamental physics is painfully lacking. Anthropic reasoning can give the illusion of intelligent design [36], without the need for any intelligent intervention.

On the other hand, many other physicists have an abhorrence of anthropic reasoning. To this group, anthropic reasoning means the end of the hope that precise and unique predictions can be made on the basis of logical deduction [37]. Since this hope should not be given up lightly, many physicists are still trying to find some mechanism to pick out a unique vacuum from string theory. So far there is no discernable progress.

It seems sensible, to me, to consider anthropic reasoning to be the explanation of last resort. That is, in the absence of any detailed understanding of the multiverse, life, or the evolution of either, anthropic arguments become plausible only when we cannot find any other explanation. That said, I find it difficult to know whether the cosmological constant problem is severe enough to justify the explanation of last resort.

Inflation can conceivably help in the search for a nonanthropic explanation of vacuum selection, since it offers the possibility that only a small minority of vacua are populated. Inflation is, after all, a complicated mechanism that involves exponentially large factors in its basic description, so it possible that it populates some states overwhelming more than others. In particular, one might expect that those states that lead to the fastest exponential expansion rates would be favored. Then these fastest expanding states — and their decay products — could dominate the multiverse.

But so far, unfortunately, this is only wishful thinking. As I will discuss in the next section, we do not even know how to define probabilities in eternally inflating multiverses. Furthermore, it does not seem likely that any principle that favors a rapid rate of exponential inflation will favor a vacuum of the type that we live in. The key problem, as one might expect, is the value of the cosmological constant. The cosmological constant Λ in our universe is extremely small, i.e., $\Lambda \lesssim 10^{-120}$ in Planck units. If inflation singles out the state with the fastest exponential expansion rate, the energy density of that state would be expected to be of order Planck scale or larger. To explain why our vacuum has such a small energy density, we would need to find some reason why this very high energy density state should decay preferentially to a state with an exceptionally small energy density [38].

There has been some effort to find relaxation methods that might pick out the vacuum [39], and perhaps this is the best hope for a nonanthropic explanation of the cosmological constant. So far, however, the landscape of nonanthropic solutions to this problem seems bleak.

4. Difficulties in Calculating Probabilities

In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times. Thus, the question of what is possible becomes trivial—anything is possible, unless it violates some absolute conservation law. To extract predictions from the theory, we must therefore learn to distinguish the probable from the improbable.

However, as soon as one attempts to define probabilities in an eternally inflating spacetime, one discovers ambiguities. The problem is that the sample space is infinite, in that an eternally inflating universe produces an infinite number of pocket universes. The fraction of universes with any particular property is therefore equal to infinity divided by infinity—a meaningless ratio. To obtain a well-defined answer, one needs to invoke some method of regularization.

To understand the nature of the problem, it is useful to think about the integers as a model system with an infinite number of entities. We can ask, for example, what fraction of the integers are odd. Most people would presumably say that the answer is 1/2, since the integers alternate between odd and even. That is, if the string of integers is truncated after the Nth, then the fraction of odd integers in the string is exactly 1/2 if N is even, and is $(N+1)/2N$ if N is odd. In any case, the fraction approaches 1/2 as N approaches infinity.

However, the ambiguity of the answer can be seen if one imagines other orderings for the integers. One could, if one wished, order the integers as

$$1, 3,\ 2,\ 5, 7,\ 4,\ 9, 11,\ 6\ , \ldots , \qquad (11)$$

always writing two odd integers followed by one even integer. This series includes each integer exactly once, just like the usual sequence $(1, 2, 3, 4, \ldots)$. The integers are just arranged in an unusual order. However, if we truncate the sequence shown in Eq. (11) after the Nth entry, and then take the limit $N \to \infty$, we would conclude that 2/3 of the integers are odd. Thus, we find that the definition of probability on an infinite set requires some method of truncation, and that the answer can depend nontrivially on the method that is used.

In the case of eternally inflating spacetimes, the natural choice of truncation might be to order the pocket universes in the sequence in which they form. However, we must remember that each pocket universe fills its own future light cone, so no pocket universe forms in the future light cone of another. Any two pocket universes are spacelike separated from each other, so some observers will see one as forming first, while other observers will see the opposite. One can arbitrarily choose equal-time surfaces that foliate the spacetime, and then truncate at some value of t, but this recipe is not unique. In practice, different ways of choosing equal-time surfaces give different results.

5. The Youngness Paradox

If one chooses a truncation in the most naive way, one is led to a set of very peculiar results which I call the *youngness paradox.*

Specifically, suppose that one constructs a Robertson-Walker coordinate system while the model universe is still in the false vacuum (de Sitter) phase, before any pocket universes have formed. One can then propagate this coordinate system forward with a synchronous gauge condition,§ and one can define probabilities by truncating at a fixed value t_f of the synchronous time coordinate t. That is, the probability of any particular property can be taken to be proportional to the volume on the $t = t_f$ hypersurface which has that property. This method of defining probabilities was studied in detail by Linde, Linde, and Mezhlumian, in a paper with the memorable title "Do we live in the center of the world?" [40]. I will refer to probabilities defined in this way as synchronous gauge probabilities.

The youngness paradox is caused by the fact that the volume of false vacuum is growing exponentially with time with an extraordinary time constant, in the vicinity of 10^{-37} s. Since the rate at which pocket universes form is proportional to the volume of false vacuum, this rate is increasing exponentially with the same time constant. That means that in each second the number of pocket universes that exist is multiplied by a factor of $\exp\{10^{37}\}$. At any given time, therefore, almost all of the pocket universes that exist are universes that formed very very recently, within the last several time constants. The population of pocket universes is therefore an incredibly youth-dominated society, in which the mature universes are vastly outnumbered by universes that have just barely begun to evolve. Although the mature universes have a larger volume, this multiplicative factor is of little importance, since in synchronous coordinates the volume no longer grows exponentially once the pocket universe forms.

Probability calculations in this youth-dominated ensemble lead to peculiar results, as discussed in Ref. [40]. These authors considered the expected behavior of the mass density in our vicinity, concluding that we should find ourselves very near the center of a spherical low-density region. Here I would like to discuss a less physical but simpler question, just to illustrate the paradoxes associated with synchronous gauge probabilities. Specifically, I will consider the question: "Are there any other civilizations

§ By a synchronous gauge condition, I mean that each equal-time hypersurface is obtained by propagating every point on the previous hypersurface by a fixed infinitesimal time interval Δt in the direction normal to the hypersurface.

in the visible universe that are more advanced than ours?". Intuitively I would not expect inflation to make any predictions about this question, but I will argue that the synchronous gauge probability distribution strongly implies that there is no civilization in the visible universe more advanced than us.

Suppose that we have reached some level of advancement, and suppose that $t_{\rm min}$ represents the minimum amount of time needed for a civilization as advanced as we are to evolve, starting from the moment of the decay of the false vacuum—the start of the big bang. The reader might object on the grounds that there are many possible measures of advancement, but I would respond by inviting the reader to pick any measure she chooses; the argument that I am about to give should apply to all of them. The reader might alternatively claim that there is no sharp minimum $t_{\rm min}$, but instead we should describe the problem in terms of a function which gives the probability that, for any given pocket universe, a civilization as advanced as we are would develop by time t. I believe, however, that the introduction of such a probability distribution would merely complicate the argument, without changing the result. So, for simplicity of discussion, I will assume that there is some sharply defined minimum time $t_{\rm min}$ required for a civilization as advanced as ours to develop.

Since we exist, our pocket universe must have an age t_0 satisfying

$$t_0 \geq t_{\rm min} \ . \tag{12}$$

Suppose, however, that there is some civilization in our pocket universe that is more advanced than we are, let us say by 1 second. In that case Eq. (12) is not sufficient, but instead the age of our pocket universe would have to satisfy

$$t_0 \geq t_{\rm min} + 1 \text{ second} \ . \tag{13}$$

However, in the synchronous gauge probability distribution, universes that satisfy Eq. (13) are outnumbered by universes that satisfy Eq. (12) by a factor of approximately $\exp\{10^{37}\}$. Thus, if we know only that we are living in a pocket universe that satisfies Eq. (12), it is extremely improbable that it also satisfies Eq. (13). We would conclude, therefore, that it is extraordinarily improbable that there is a civilization in our pocket universe that is at least 1 second more advanced than we are.

Perhaps this argument explains why SETI has not found any signals from alien civilizations, but I find it more plausible that it is merely a symptom that the synchronous gauge probability distribution is not the right one.

Although the problem of defining probabilities in eternally inflating universe has not been solved, a great deal of progress has been made in exploring options and understanding their properties. For many years Vilenkin and his collaborators [28, 41] were almost the only cosmologists working on this issue, but now the field is growing rapidly [42].

6. Does Inflation Need a Beginning?

If the universe can be eternal into the future, is it possible that it is also eternal into the past? Here I will describe a recent theorem [43] which shows, under plausible assumptions, that the answer to this question is no.‖

The theorem is based on the well-known fact that the momentum of an object traveling on a geodesic through an expanding universe is redshifted, just as the momentum of a photon is redshifted. Suppose, therefore, we consider a timelike or null geodesic extended backwards, into the past. In an expanding universe such a geodesic will be blueshifted. The theorem shows that under some circumstances the blueshift reaches infinite rapidity (i.e., the speed of light) in a finite amount of proper time (or affine parameter) along the trajectory, showing that such a trajectory is (geodesically) incomplete.

To describe the theorem in detail, we need to quantify what we mean by an expanding universe. We imagine an observer whom we follow backwards in time along a timelike or null geodesic. The goal is to define a local Hubble parameter along this geodesic, which must be well-defined even if the spacetime is neither homogeneous nor isotropic. Call the velocity of the geodesic observer $v^\mu(\tau)$, where τ is the proper time in the case of a timelike observer, or an affine parameter in the case of a null observer. (Although we are imagining that we are following the trajectory backwards in time, τ is defined to increase in the future timelike direction, as usual.) To define H, we must imagine that the vicinity of the observer is filled with "comoving test particles," so that there is a test particle velocity $u^\mu(\tau)$ assigned to each point τ along the geodesic trajectory, as shown in Fig. 5. These particles need not be real — all that will be necessary is that the worldlines can be defined, and that each worldline should have zero proper acceleration at the instant it intercepts the geodesic observer.

To define the Hubble parameter that the observer measures at time τ, the observer focuses on two particles, one that he passes at time τ, and one at $\tau + \Delta\tau$, where in the end he takes the limit $\Delta\tau \to 0$. The Hubble parameter is defined by

$$H \equiv \frac{\Delta v_{\rm radial}}{\Delta r} \ , \tag{14}$$

where $\Delta v_{\rm radial}$ is the radial component of the relative velocity between the two particles, and Δr is their distance, where both quantities are computed in the rest frame of one of the test particles, not in the rest frame of the observer. Note that this definition reduces to the usual one if it is applied to a homogeneous isotropic universe.

The relative velocity between the observer and the test particles can be measured by the invariant dot product,

$$\gamma \equiv u_\mu v^\mu \ , \tag{15}$$

‖ There were also earlier theorems about this issue by Borde and Vilenkin (1994, 1996) [44, 45], and Borde [46] (1994), but these theorems relied on the weak energy condition, which for a perfect fluid is equivalent to the condition $\rho + p \geq 0$. This condition holds classically for forms of matter that are known or commonly discussed as theoretical proposals. It can, however, be violated by quantum fluctuations [47], and so the applicability of these theorems is questionable.

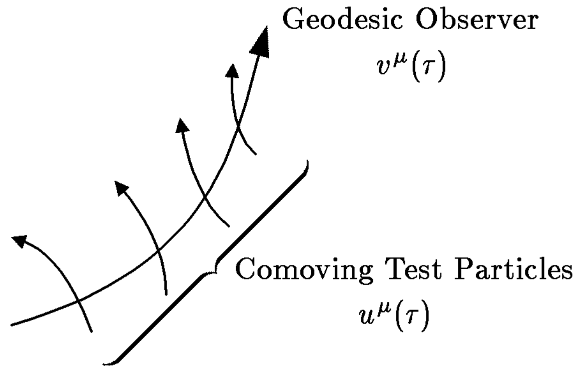

Figure 5. An observer measures the velocity of passing test particles to infer the Hubble parameter.

which for the case of a timelike observer is equal to the usual special relativity Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - v_{\rm rel}^2}} \ . \qquad (16)$$

If H is positive we would expect γ to decrease with τ, since we expect the observer's momentum relative to the test particles to redshift. It turns out, however, that the relationship between H and changes in γ can be made precise. If one defines

$$F(\gamma) \equiv \begin{cases} 1/\gamma & \text{for null observers} \\ \operatorname{arctanh}(1/\gamma) & \text{for timelike observers} \end{cases} , \qquad (17)$$

then

$$H = \frac{dF(\gamma)}{d\tau} \ . \qquad (18)$$

I like to call $F(\gamma)$ the "slowness" of the geodesic observer, because it increases as the observer slows down, relative to the test particles. The slowness decreases as we follow the geodesic backwards in time, but it is positive definite, and therefore cannot decrease below zero. $F(\gamma) = 0$ corresponds to $\gamma = \infty$, or a relative velocity equal to that of light. This bound allows us to place a rigorous limit on the integral of Eq. (18). For timelike geodesics,

$$\int^{\tau_f} H \, d\tau \le \operatorname{arctanh}\left(\frac{1}{\gamma_f}\right) = \operatorname{arctanh}\left(\sqrt{1 - v_{\rm rel}^2}\right) \ , \qquad (19)$$

where γ_f is the value of γ at the final time $\tau = \tau_f$. For null observers, if we normalize the affine parameter τ by $d\tau/dt = 1$ at the final time τ_f, then

$$\int^{\tau_f} H \, d\tau \le 1 \ . \qquad (20)$$

Thus, if we assume an *averaged expansion condition*, i.e., that the average value of the Hubble parameter $H_{\rm av}$ along the geodesic is positive, then the proper length (or affine length for null trajectories) of the backwards-going geodesic is bounded. Thus the region for which $H_{\rm av} > 0$ is past-incomplete.

It is difficult to apply this theorem to general inflationary models, since there is no accepted definition of what exactly defines this class. However, in standard eternally inflating models, the future of any point in the inflating region can be described by a stochastic model [48] for inflaton evolution, valid until the end of inflation. Except for extremely rare large quantum fluctuations, $H \gtrsim \sqrt{(8\pi/3)G\rho_f}$, where ρ_f is the energy density of the false vacuum driving the inflation. The past for an arbitrary model is less certain, but we consider eternal models for which the past is like the future. In that case H would be positive almost everywhere in the past inflating region. If, however, $H_{\rm av} > 0$ when averaged over a past-directed geodesic, our theorem implies that the geodesic is incomplete.

There is of course no conclusion that an eternally inflating model must have a unique beginning, and no conclusion that there is an upper bound on the length of all backwards-going geodesics from a given point. There may be models with regions of contraction embedded within the expanding region that could evade our theorem. Aguirre and [49, 50] have proposed a model that evades our theorem, in which the arrow of time reverses at the $t = -\infty$ hypersurface, so the universe "expands" in both halves of the full de Sitter space.

The theorem does show, however, that an eternally inflating model of the type usually assumed, which would lead to $H_{\rm av} > 0$ for past-directed geodesics, cannot be complete. Some new physics (i.e., not inflation) would be needed to describe the past boundary of the inflating region. One possibility would be some kind of quantum creation event.

One particular application of the theory is the cyclic ekpyrotic model of Steinhardt & Turok [51]. This model has $H_{\rm av} > 0$ for null geodesics for a single cycle, and since every cycle is identical, $H_{\rm av} > 0$ when averaged over all cycles. The cyclic model is therefore past-incomplete, and requires a boundary condition in the past.

7. Conclusion

In this paper I have summarized the arguments that strongly suggest that our universe is the product of inflation. I argued that inflation can explain the size, the Hubble expansion, the homogeneity, the isotropy, and the flatness of our universe, as well as the absence of magnetic monopoles, and even the characteristics of the nonuniformities. The detailed observations of the cosmic background radiation anisotropies continue to fall in line with inflationary expectations, and the evidence for an accelerating universe fits beautifully with the inflationary preference for a flat universe. Our current picture of the universe seems strange, with 95% of the energy in forms of matter that we do not understand, but nonetheless the picture fits together extraordinarily well.

Next I turned to the question of eternal inflation, claiming that essentially all inflationary models are eternal. In my opinion this makes inflation very robust: if it starts anywhere, at any time in all of eternity, it produces an infinite number of pocket universes. A crucial issue in our understanding of fundamental physics is the selection of the vacuum, which according to current ideas in string theory could be any one of a colossal number of possibilities. Eternal inflation offers at least a hope that a small set of vacua might be strongly favored. For that reason it is important for us to learn more about the evolution of the multiverse during eternal inflation. But so far it is only wishful thinking to suppose that eternal inflation will allow us to determine the vacuum in which we should expect to find ourselves.

I then discussed the past of eternally inflating models, concluding that under mild assumptions the inflating region must have a past boundary, and that new physics (other than inflation) is needed to describe what happens at this boundary.

Although eternal inflation has fascinating consequences, our understanding of it remains incomplete. In particular, we still do not understand how to define probabilities in an eternally inflating spacetime.

We should keep in mind, however, that observations in the past few years have vastly improved our knowledge of the early universe, and that these new observations have been generally consistent with the simplest inflationary models. It is the success of these predictions that justifies spending time on the more speculative aspects of inflationary cosmology.

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[20] I thank Max Tegmark for providing this graph, an earlier version of which appeared in Ref. [21]. The graph shows the most precise data points for each range of ℓ from recent observations, as summarized in Refs. [5] and [22]. The cosmic string prediction is taken from Ref. [23], and the "Inflation with Λ" curve was calculated from the best-fit parameters to the WMAP 3-year data from Table 5 of Ref. [22]. The other curves were both calculated for $n_s = 1$, $\Omega_{\rm baryon} = 0.05$, and $H = 70$ km s^{-1} Mpc^{-1}, with the remaining parameters fixed as follows. "Inflation without Λ": $\Omega_{\rm DM} = 0.95$, $\Omega_\Lambda = 0$, $\tau = 0.06$; "Open universe": $\Omega_{\rm DM} = 0.25$, $\Omega_\Lambda = 0$, $\tau = 0.06$. With our current ignorance of the underlying physics, none of these theories predicts the overall amplitude of the fluctuations; the "Inflation with Λ" curve was normalized for a best fit, and the others were normalized arbitrarily.

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